Emergent scale symmetry: Connecting inflation and dark energy

Dr. Javier Rubio

Universität Heidelberg
Zukunft. Seit 1386.

The scaling frame

All dimensionless couplings and masses in the theory are allowed to depend on the expectation value of a scalar field χ

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{\chi^2}{2}\tilde{R} - \frac{B(\chi/\mu) - 6}{2}(\tilde{\partial}\chi)^2 - \mu^2\chi^2$$

The only fixed scale not proportional to χ is the scale μ, which is associated to the dilatation anomaly. The value of μ has no intrinsic meaning and can be used to set the mass/time scales

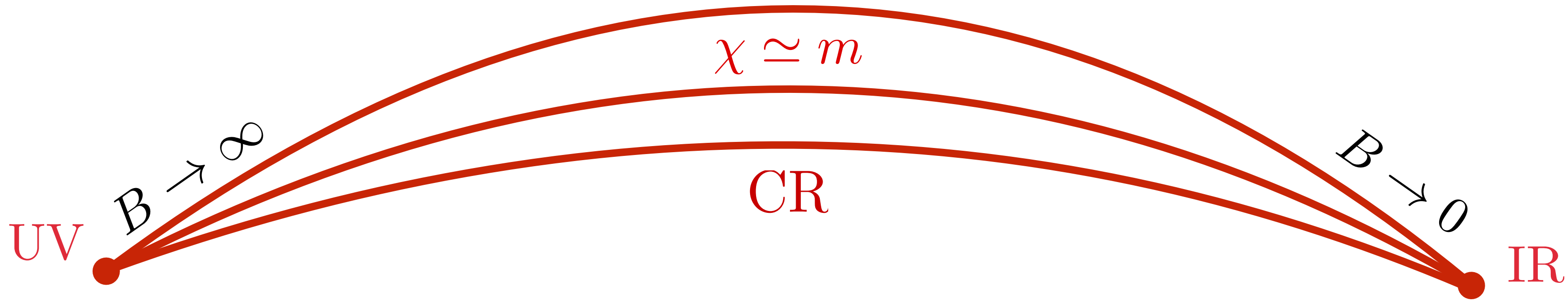
$$\mu^{-1} = 10^{10}\,\mathrm{yr} = 1.2 \times 10^{60}\, M_P^{-1}$$

For $\mu = 0$ and constant B the action is dilatation invariant

For a canonical scaling of χ, $\mu^2\chi^2$ spoils scale invariance.

The anomalous dimension of χ in the UV is crucial

Fixed point structure



$$\mu \partial_\mu B = \sigma B$$

$$B \simeq \left(\frac{m}{\chi}\right)^{\sigma}$$

$$\mu \partial_\mu B = \kappa B^2$$

$$B \simeq \frac{1}{\kappa \log(\chi/m)}$$

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$\frac{\sigma}{\kappa B(\chi)} = \mathcal{W}\left[\frac{\sigma}{\kappa}\left(\frac{\chi}{m}\right)^{\sigma}\right] \quad \text{with} \quad m \equiv \mu \exp(c_t)$$

c_t selects the particular trajectory in the flow

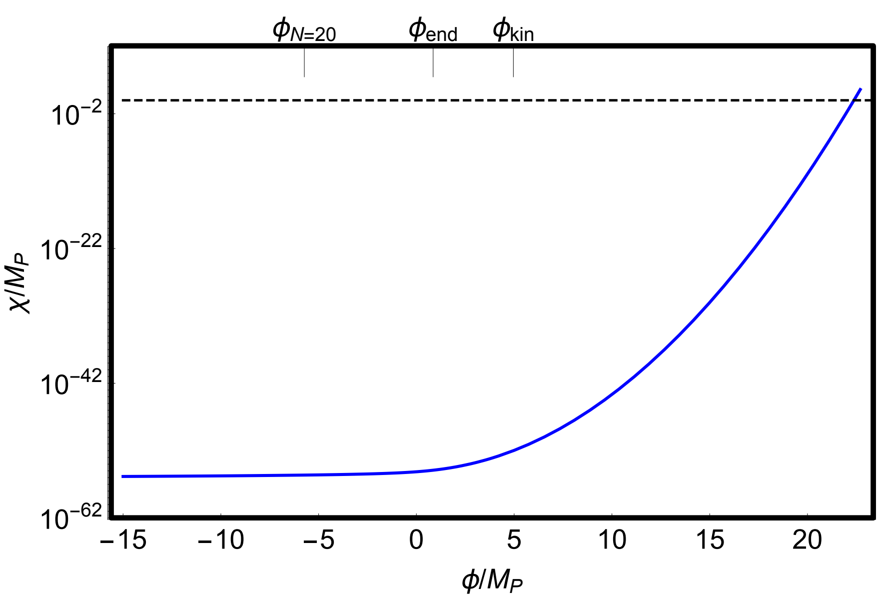
3 parameters

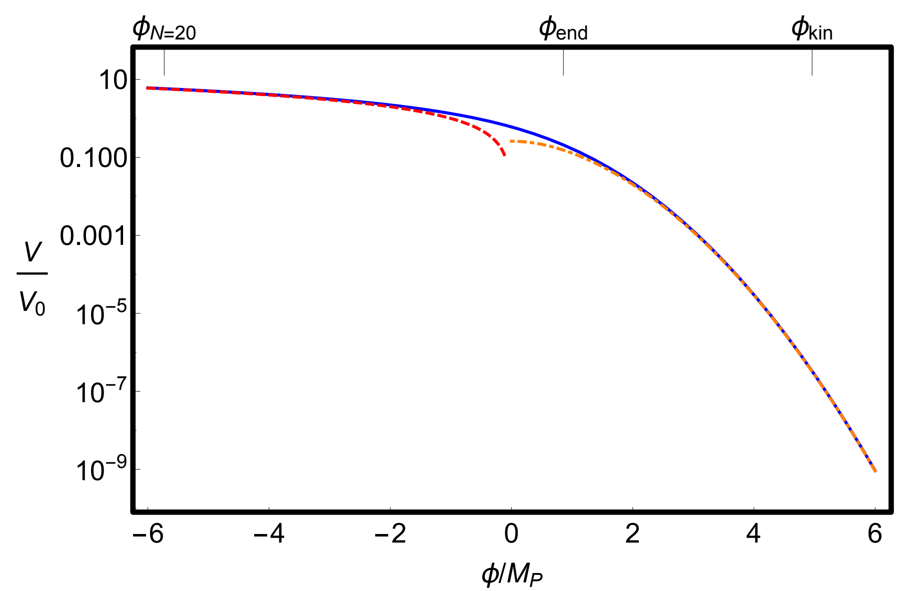
Einstein-frame formulation

$$V = \left(\frac{\mu}{\chi}\right)^2 = V_0 \left[\frac{\exp\left(-Y\right)}{Y}\right]^{2/\sigma} \quad \text{with} \quad V_0 = \left(\frac{\mu}{m}\right)^2 \left(\frac{\sigma}{\kappa}\right)^{2/\sigma}$$

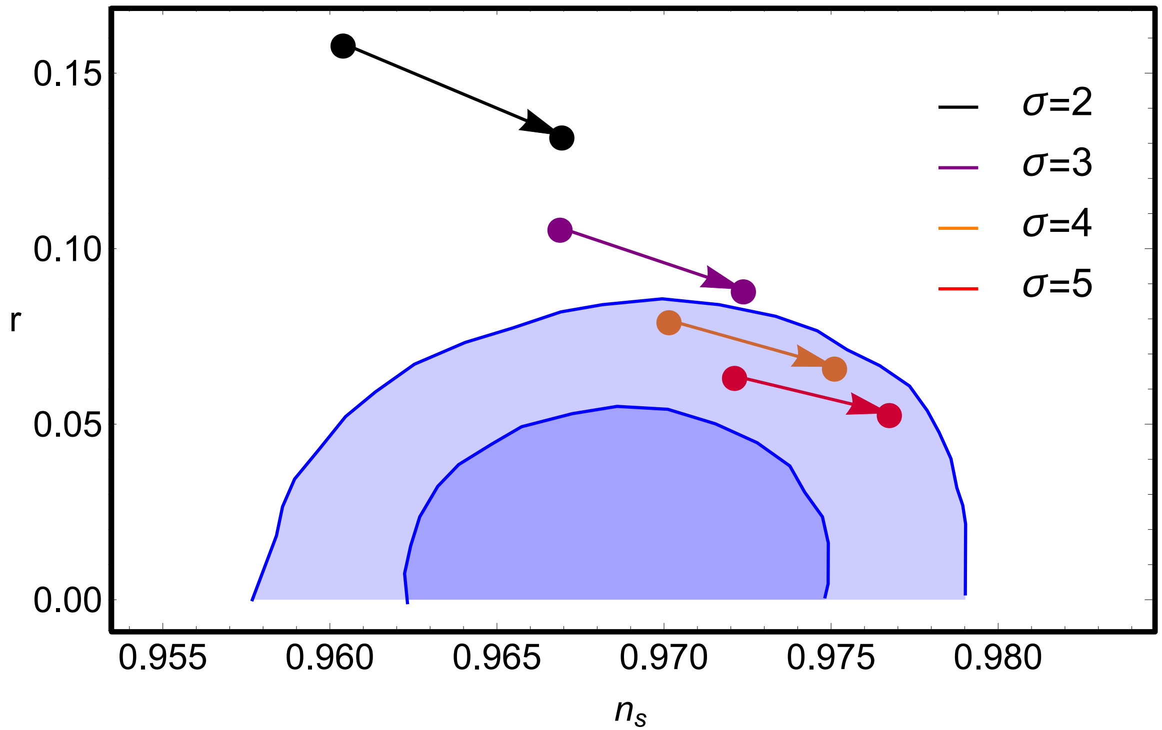
The function Y is asymmetric

$$Y = \frac{\sigma}{\kappa B(\chi)} = 1 + \frac{1}{2}\left[\frac{\phi^2}{\phi_t^2} + \frac{\phi}{\phi_t}\sqrt{4 + \frac{\phi^2}{\phi_t^2}}\right] \quad \text{with} \quad \phi_t = \frac{2M_P}{\sqrt{\kappa\sigma}}$$





The inflationary era $V(\phi) \simeq V_0 \left(\frac{\phi^2}{\phi_t^2}\right)^{2/\sigma}$



Scalar spectral tilt

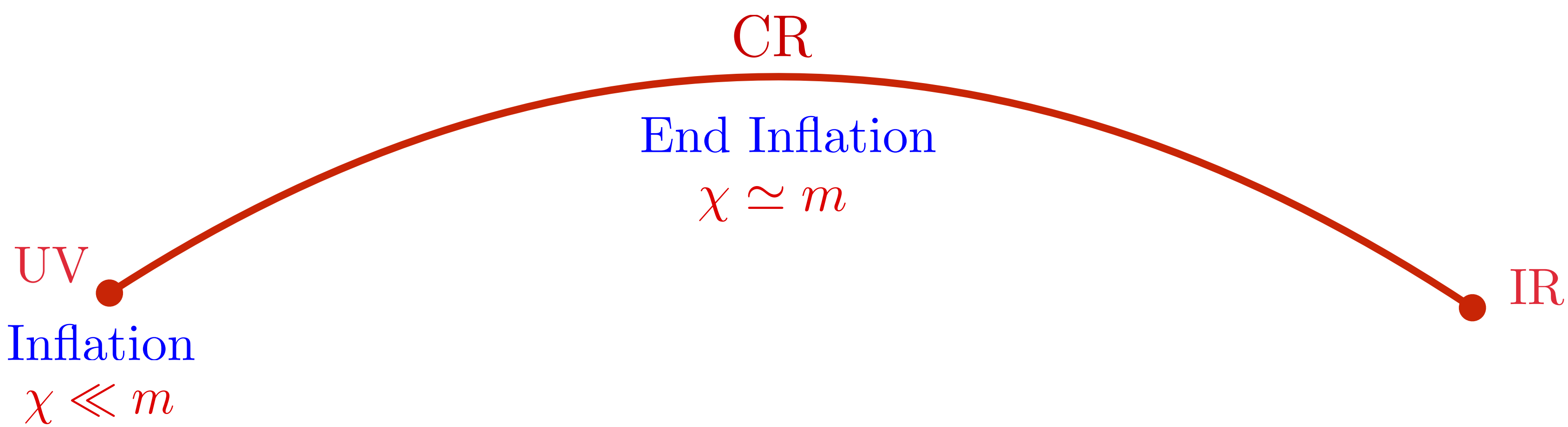
$$1 - n_s = \frac{2+\sigma}{\sigma N + 1}$$

Tensor-to-scalar ratio

$$r = \frac{16}{\sigma N + 1}$$

A simple overall picture emerges

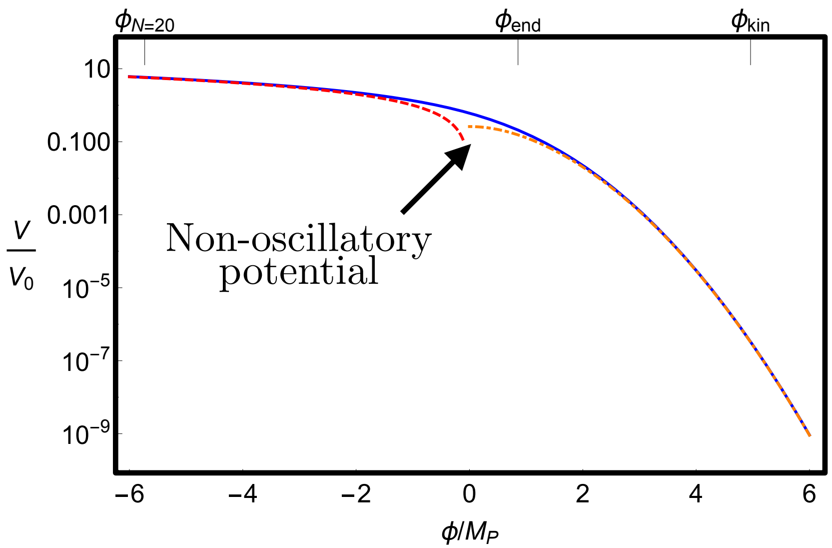
The approximate scale symmetry emerging at the UV fixed point is the origin of the almost flat primordial fluctuation spectrum.

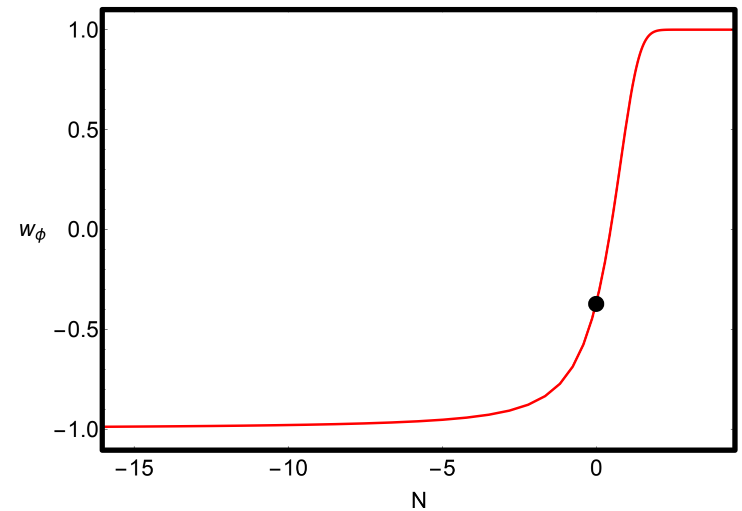


The scale m is an integration constant of an almost logarithmic flow. The small amplitude of scalar perturbations arises naturally.

$$\mathcal{A} = \frac{1}{32}\left[2(\sigma N + 1)\right]^{1+\frac{2}{\sigma}} \frac{\mu^2}{m^2} \propto \exp(-2c_t)$$

The kinetic dominated era





For hBB to begin, the energy must be transmitted to the SM d.o.f.

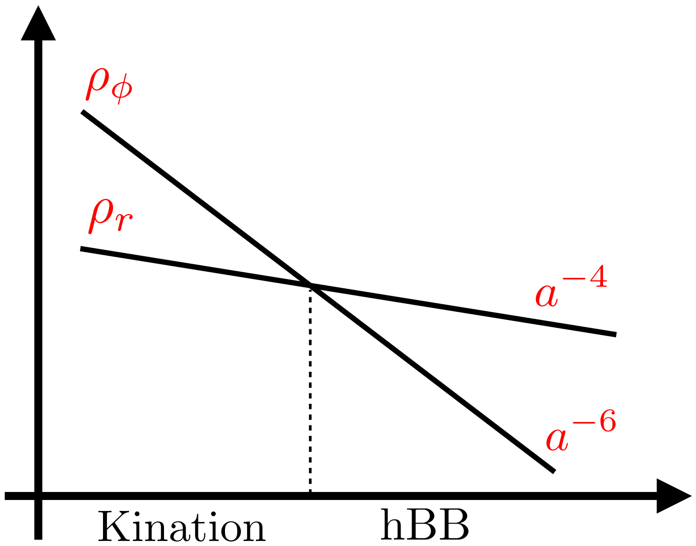
Two requirements

1) Kinetic domination has to end before Big Bang Nucleosynthesis.

2) The GW density fraction at BBN should not be too large

$$\Omega_{\rm GW}(k) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k} \propto k^{2\left(\frac{3w-1}{3w+1}\right)} \qquad h^2\int_{k_{\rm BBN}}^{k_{\rm end}} \Omega_{\rm GW}(k)\, d\ln k \lesssim 10^{-5}$$

Heating the Universe



1) Gravitational interactions

2) Matter interactions

Heating efficiency

$$\Theta \equiv \frac{\rho_{\rm r}^{\rm kin}}{\rho_\phi^{\rm kin}} = \left(\frac{a_{\rm kin}}{a_{\rm rad}}\right)^2$$

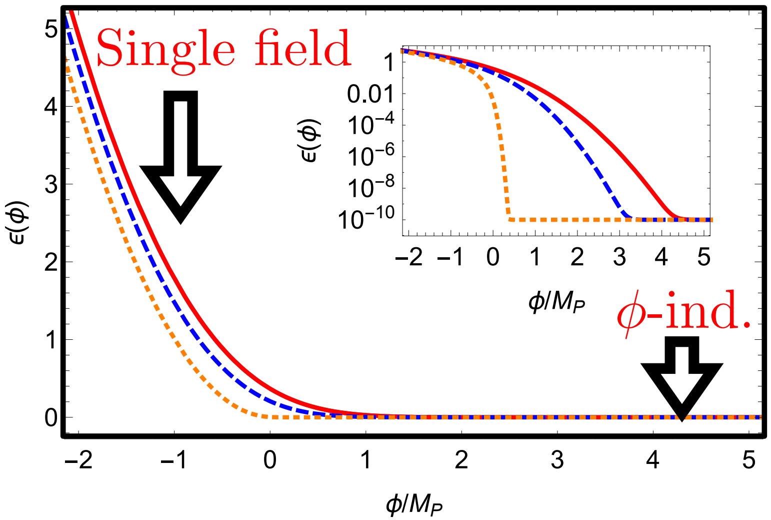
Radiation temperature

$$\rho_{\rm r}^{\rm rad} = \rho_\phi^{\rm rad}$$

$$T_{\rm rad} \equiv \left(\frac{30\,\rho_{\rm r}^{\rm rad}}{\pi^2 g_*^{\rm rad}}\right)^{1/4}$$

Direct matter couplings

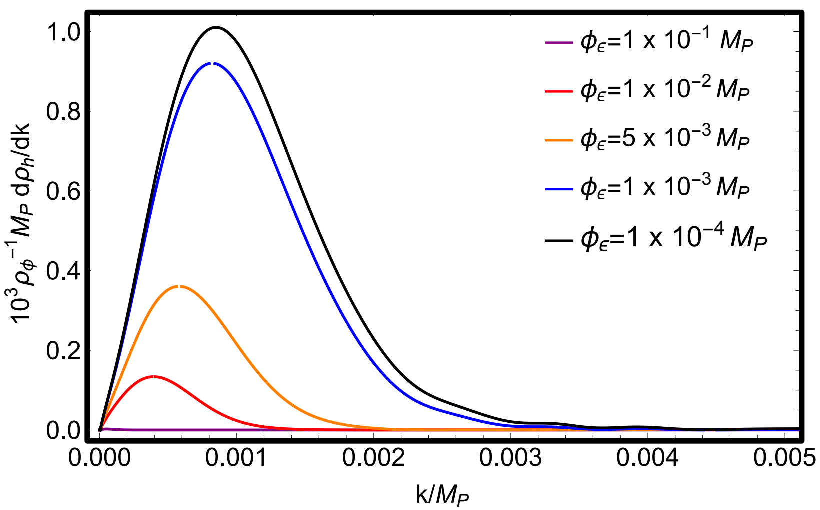
$$\frac{\mathcal{L}_{\rm I}}{\sqrt{-g}} = -\frac{1}{2}\left[(\partial h)^2 + \gamma(\phi)(\partial h^2\,\partial\phi) + M_h^2(\phi)h^2\right]$$



A possible parametrization is $M_h^2(\phi) \equiv \epsilon(\phi) M_P^2$

$$\epsilon(\phi) = \epsilon_\infty + \epsilon_1\left[\frac{\exp\left(-Y_\epsilon(\phi)\right)}{Y_\epsilon(\phi)}\right]^{\sigma_h/2}$$

$$Y_\epsilon(\phi) = 1 + \frac{1}{2}\left[\frac{\phi^2}{\phi_\epsilon^2} + \frac{\phi}{\phi_\epsilon}\sqrt{4 + \frac{\phi^2}{\phi_\epsilon^2}}\right]$$



ϕ_ϵ/M_P	Θ	$(g_*^{\rm rad})^{\frac{1}{4}} T_{\rm rad}$ (GeV)
1×10^{-1}	2.5×10^{-9}	3.9×10^{8}
1×10^{-2}	9.7×10^{-8}	6.1×10^{9}
5×10^{-3}	3.3×10^{-7}	1.5×10^{10}
1×10^{-3}	1.2×10^{-6}	4.0×10^{10}
1×10^{-4}	1.4×10^{-6}	4.4×10^{10}

The hot Big Bang era

$$H^2 = \frac{1}{3M_P^2}\left(\rho_\phi + \rho_R + \rho_M\right)$$

$$\dot{H} = -\frac{1}{2M_P^2}\left(\dot{\phi}^2 + \frac{4}{3}\rho_R + \rho_M\right)$$

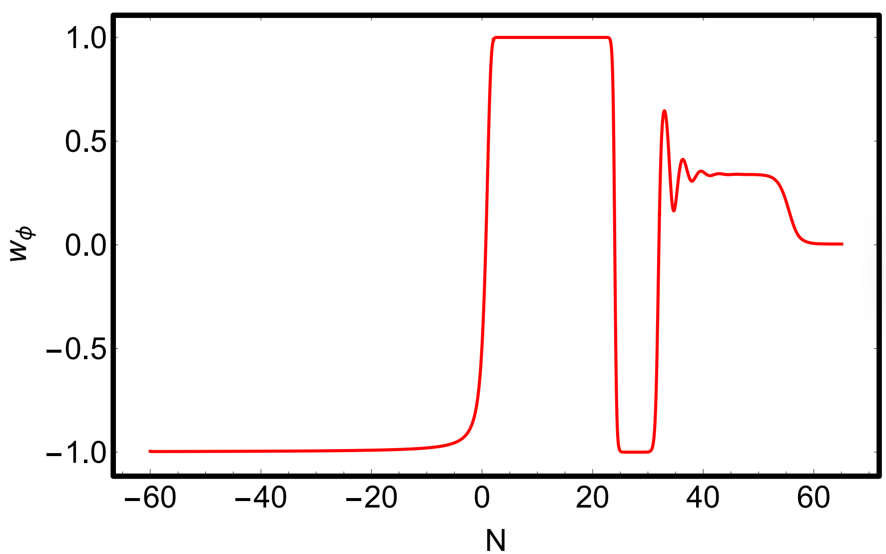
$$\dot{\rho} + nH\dot{\rho} = 0$$

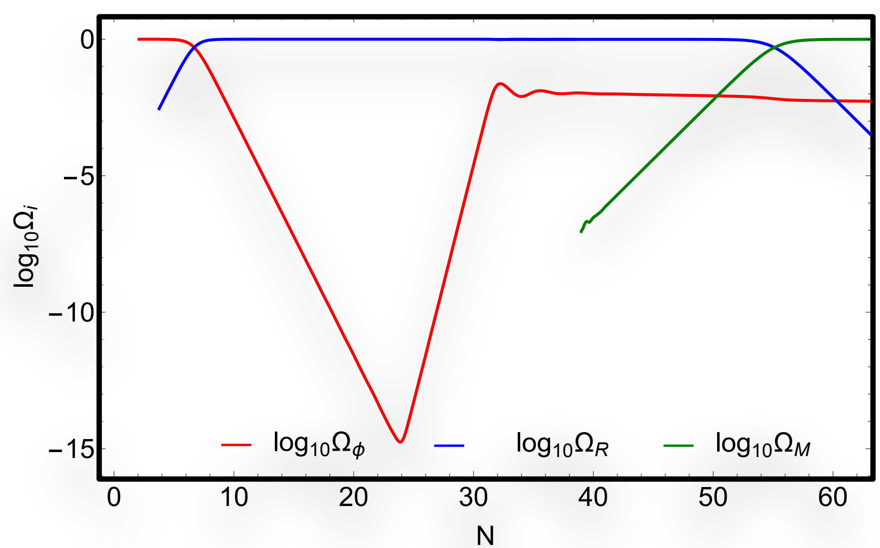
$$\ddot{\phi} + 3H\dot{\phi} + M_P^4 V_{,\phi} = 0$$

with

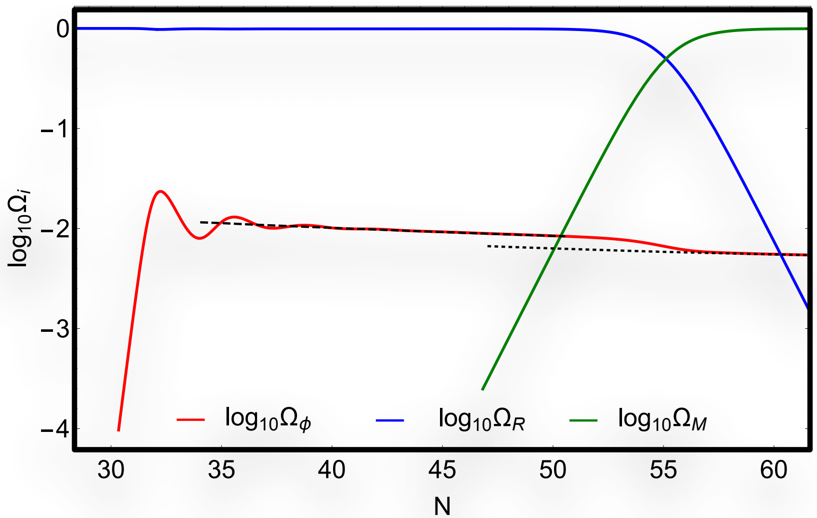
$$\rho = \rho_R + \rho_M$$

$$V(\phi) \simeq V_0 \left[\frac{\exp\left(-\frac{\phi^2}{\phi_t^2}\right)}{\frac{\phi^2}{\phi_t^2}}\right]^{\frac{2}{\sigma}}$$





Scaling solutions



$$\frac{w'_\phi}{1-w_\phi} = -3(1+w_\phi) + \lambda\sqrt{3(1+w_\phi)\Omega_\phi}$$

$$\Omega'_\phi = -\Omega_\phi(1-\Omega_\phi)\left(3(1+w_\phi)-n\right)$$

$$\lambda' = -\sqrt{3(1+w_\phi)\Omega_\phi}(\Gamma-1)\lambda^2$$

$$\lambda \equiv -M_{\rm P}\frac{V_{,\phi}}{V}\,, \qquad \Gamma \equiv \frac{VV_{,\phi\phi}}{V_{,\phi}^2}$$

A simple inspection reveals, for constant λ, a fixed point at

$$\Omega_\phi = \frac{n}{\lambda} \qquad w_\phi = \frac{n-3}{3}$$

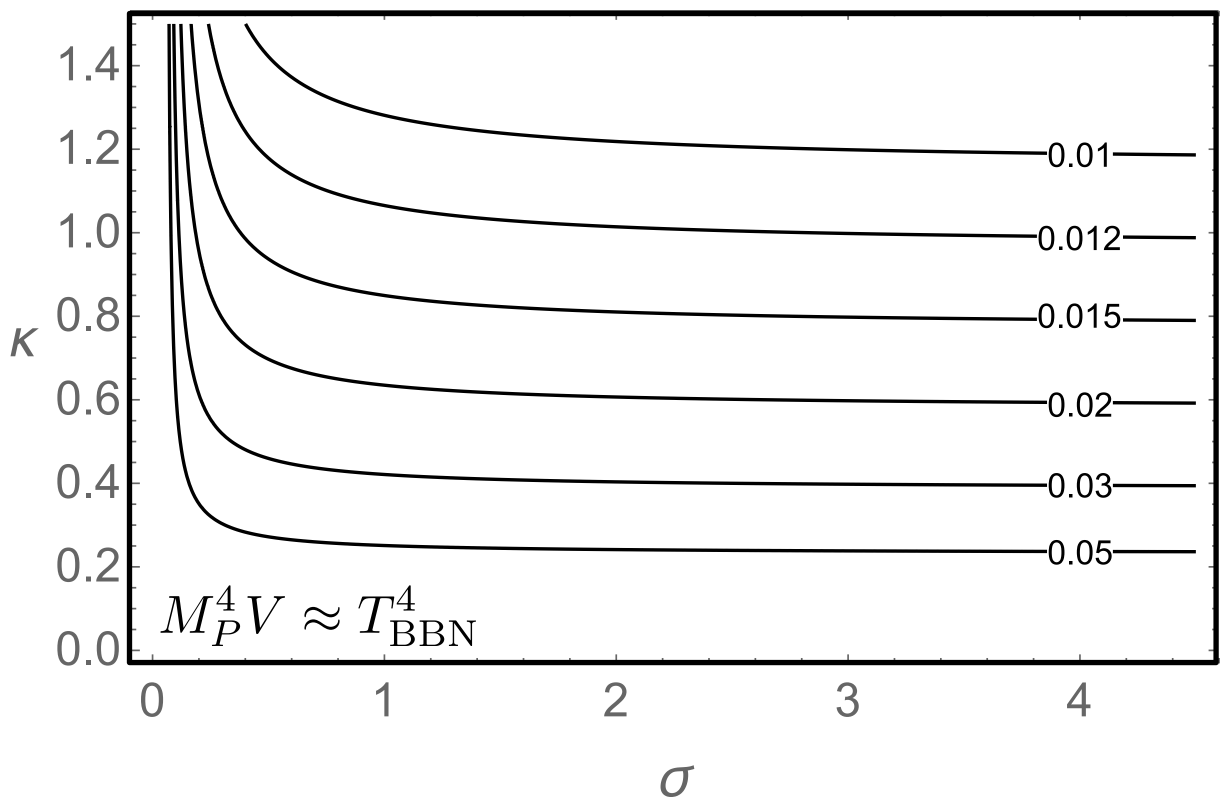
In our case

$$\lambda = 2\sqrt{\frac{\kappa Y}{\sigma}} \qquad \Gamma = 1 - \frac{\sigma}{4(1+Y)} \longrightarrow 1$$

An approximately fixed trajectory with slowly varying λ

Early DE constraints

$$\Omega_\phi = \frac{n\sigma}{4\kappa Y(\phi)} = \frac{nB(\phi(\chi))}{4}$$



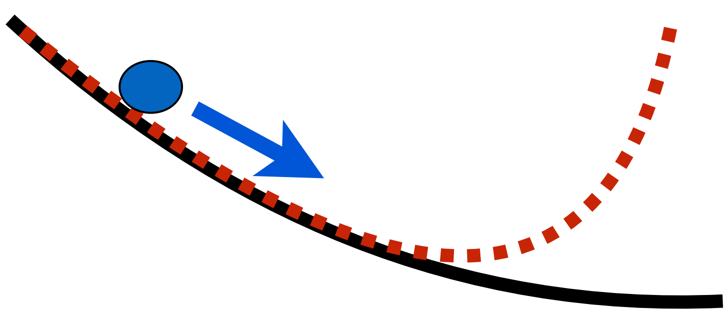
Exiting the scaling regime

This effect can be induced by a second crossover stage in the beyond the Standard Model sector which manifests itself through the non-renormalizable neutrino mass operator.

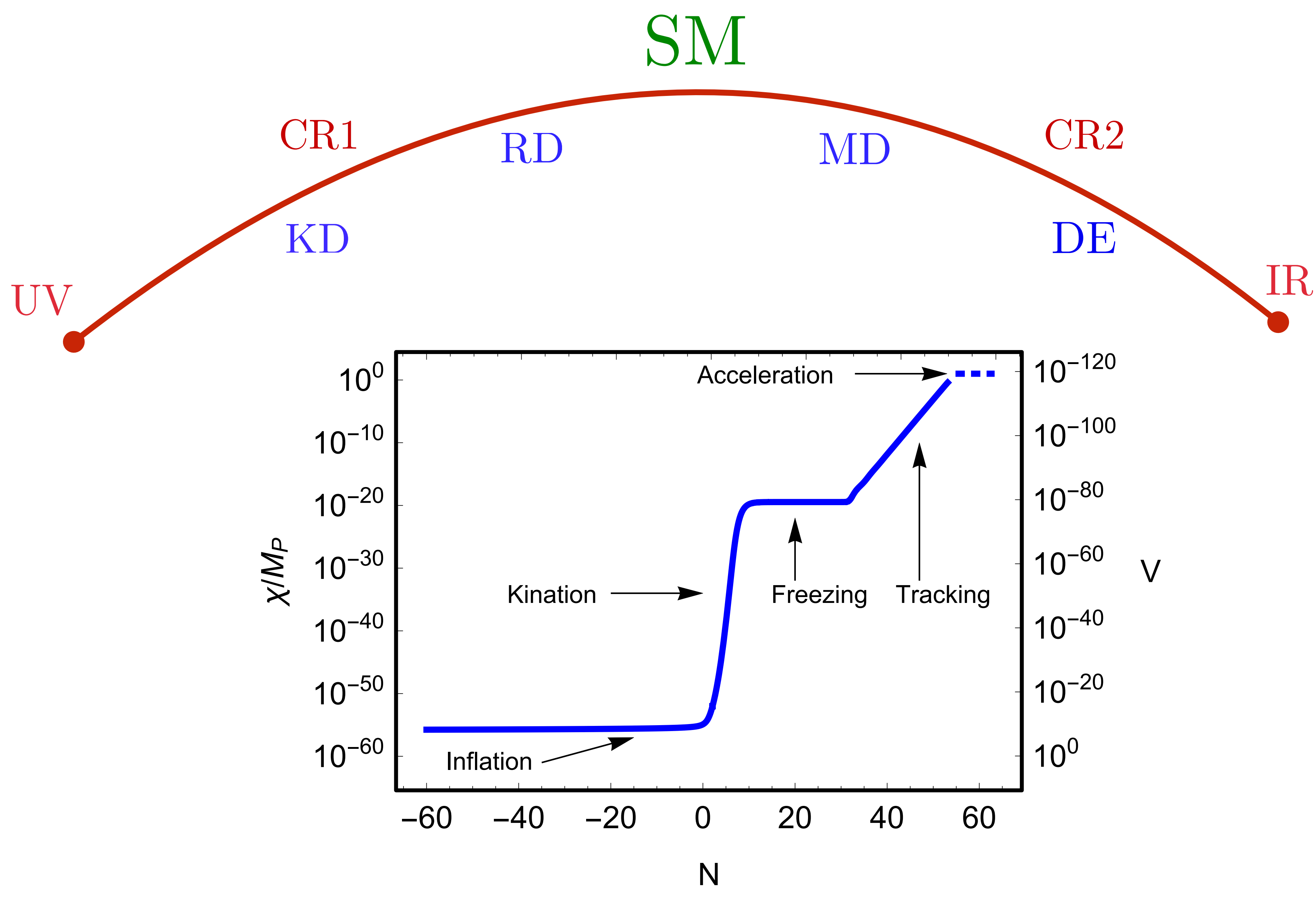
ν as clock

As soon as neutrinos become non-relativistic, the ϕ-dependence of the neutrino mass generates an additional term in the field equation for the cosmon field which effectively stops the evolution of ϕ, ending the scaling solution and leading to a cosmology that looks rather close to a cosmological constant afterwards.

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \beta(\phi)(\rho_\nu - 3p_\nu)$$



Rich dynamics from simple principles




Thanks!