Preheating in the lattice

Fabian Przybilla

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In the following pages we will take a qualitative look at how the universe looks after inflation. This report will also expand on the effect of the backreaction within the regime of preheating. Additionally we will examine some computed results of the evolution of the inflaton field and an additional scalar field in a simplified model for preheating and reheating.

1 Introduction

Starting off by looking at the universe after inflation ended, we are left with a cold and empty universe in contrast to our present hot universe. Obviously between then and now particle production had to occur, as well as a reheating of the universe. For the stage after the inflation, perturbation theory was used to describe the effects of the aptly called "reheating".

We will consider a simplified model for inflation which contains all the important physical effects like $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - g^2\phi^2\chi^2$. The basic premise of this theory is, that the inflaton field ϕ has oscillations, which will decay into χ particles, due to the interaction term. At some point the the scalar particles χ interact with themselves and soon create a thermal equilibrium with a particular reheating temperature T_r. But it was found out, that important non-perturbative effects were left out in this theory and a stage of parametric resonance had to occur beforehand: "preheating". Therefore the particle production in the presence of a strong background field ϕ was analyzed.

In the next section we will see that through backreaction the effect of preheating can be further divided into two stages. Furthermore I computed several diagrams for the preheating and reheating after inflation to serve as a visual guide to all the processes involved. For the computation of the data I used the LATTICEEASY program [3] with a simple model to get diagrams which contain the main effects of preheating and reheating clearly. These diagrams will be explained in the last section of this report.

2 Backreaction

Preheating starts with the production of χ particles, which we expect to be created exponentially since the growth itself depends on the number of particles already produced. After a certain time into preheating it is possible for these χ-particles to create ϕ-particles. This effect is called backreaction and in our next step we will examine at which point the backreaction becomes relevant to the calculations of preheating. This way the preheating can be split into two stages: with and without backreaction.

Using the Hartree approximation we can introduce a new term depending on the scalar field χ to the equation of motion of the inflaton field ϕ,

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + g^2\langle\chi^2\rangle\phi = 0, \tag{1}$$

with the following vacuum expectation value:

$$\langle\chi^2\rangle = \frac{1}{2\pi^2 a^3}\int_0^\infty \mathrm{d}k\, k^2 |X_k(t)|^2. \tag{2}$$

These amplified quantum fluctuations also affect the effective mass m_ϕ:

$$m_\phi^2 = m^2 + g^2\langle\chi^2\rangle. \tag{3}$$

In the next step we will rewrite $\langle\chi^2\rangle$ in terms of the Bogoliubov coefficients $\alpha_k(t)$ and $\beta_k(t)$,

$$\langle\chi^2\rangle = \frac{1}{2\pi^2 a^3}\int_0^\infty \frac{\mathrm{d}k\, k^2}{\omega}\left(|\beta_k|^2 + Re\left(\alpha_k\beta_k^* e^{-2i\int_0^t \omega\,\mathrm{d}t}\right)\right), \tag{4}$$

where we know that we can relate the β_k-coefficient to the occupation number and estimate its value like $|\beta_k|^2 \equiv n_k \approx \frac{1}{2}e^{2\mu_k mt}$.

Before we can further simplify the expectation value of χ^2, we have to look whether the integral needs renormalization. Since we introduced the Bogoliubov coefficients in terms of the particle production and not vacuum polarization, the integral in equation (4) is finite and we can continue without renormalization. Further we have $Re\left(\alpha_k\beta_k^* e^{-2i\int_0^t \omega\,\mathrm{d}t}\right) \approx |\beta_k|^2 \cos\left(2\int_0^t \omega \mathrm{d}t - \arg\alpha_k + \arg\beta_k\right)$ and with $\omega \approx g\phi(t) = g\Phi \sin mt$ we arrive at the following expression:

$$\langle\chi^2\rangle = \frac{1 + C\cos\frac{2g\Phi\cos mt}{m}}{2\pi^2 a^3}\int_0^\infty \frac{\mathrm{d}k\, k^2}{\omega} n_k, \tag{5}$$

where $C < 1$ is a simple factor which accounts for a small correction. All these estimates were mainly allowed because we consider resonant particle creation of χ during backreaction, as we are still in the preheating regime. For the majority of the time within this regime we are looking at broad resonance ($\phi > \phi^*$) which leads us to $\frac{k}{a} \ll g\phi$ and $\omega \approx g|\phi(t)|$. Therefore the integral in equation (5) simplifies to a factor of $\frac{n_\chi}{g|\phi(t)|}$, where we used the formula for a vacuum expectation value:

$$n_\chi = \int_0^\infty \frac{\mathrm{d}k\, k^2}{2\pi^2 a^3} n_k, \tag{6}$$

The resulting equation for the vacuum expectation value for χ^2 is

$$\langle \chi^2 \rangle \approx \left(1 + C \cos \frac{2g\Phi \cos mt}{m}\right) \frac{n_\chi}{g|\phi(t)|}. \tag{7}$$

Further we can insert this equation into the effective mass of the background field ϕ:

$$m_\phi^2 = m^2 + \left(1 + C \cos \frac{2g\Phi \cos mt}{m}\right) \frac{g n_\chi}{|\phi(t)|} \tag{8}$$

Due to the backreaction, the effective mass oscillates now with two different frequencies. The first frequency with a value of $2m$ belongs to the oscillation of $|\phi(t)|$. Additionally the second term in the brackets leads to an oscillation of m_ϕ^2 as long as $cos(mt)$ doesn't vanish ($\phi(t) \neq \Phi$). This second oscillation has a very high frequency $2g\Phi \gg m$ which leads to a very small impact on the evolution of the effective mass. After looking at the effective mass, we can obviously insert $\langle \chi^2 \rangle$ into the equation of motion as well and make the same observations about the oscillations:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \left(1 + C \cos \frac{2g\Phi \cos mt}{m}\right) \frac{g n_\chi \phi}{|\phi|} = 0 \tag{9}$$

Before dismissing the fast oscillating term, we have to consider particle production arising from this term as this quasi-periodic change breaks the adiabaticity condition. This will not lead to parametric resonance, so we will neglect the arising particle production, as well as the term itself in first approximation:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \frac{g n_\chi \phi}{|\phi|} = 0 \tag{10}$$

Also we know that $H \ll m$ holds at the end of the first stage of preheating (just before backreaction) which lets us drop the H-term. The final step in evaluating when backreaction becomes significant, is to find when the last term has about the same magnitude as the mass-term. The resulting equation shows us at which value of the number density of the χ-particles the backreaction stops being negligible:

$$n_k \simeq \frac{m^2 \Phi}{g} = \frac{2m^3}{g^2} q^{1/2} \tag{11}$$

3 Lattice-Simulation

In order to get the following graphics, I used the LATTICEEASY program [3], which was designed to compute the evolution of interacting scalar fields. Especially it is possible to simulate the effects of preheating after inflation. As for initial values of the scalar field the program uses random noise, because these values should be negligible compared to the exponential growth we expect in this simulation. It should also be mentioned that the following diagrams do not include the effects of the expansion of the universe on these values, thus we are looking at the raw interactions between the fields and effects like backreaction.

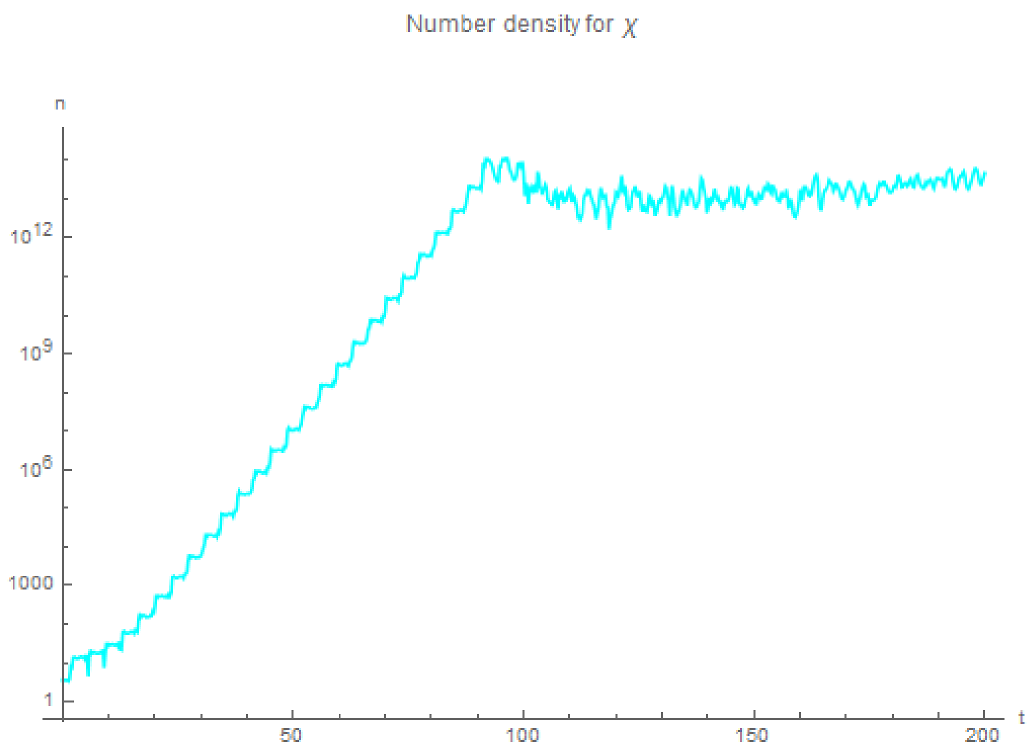

Figure 1: Evolution of the number density of the χ field.

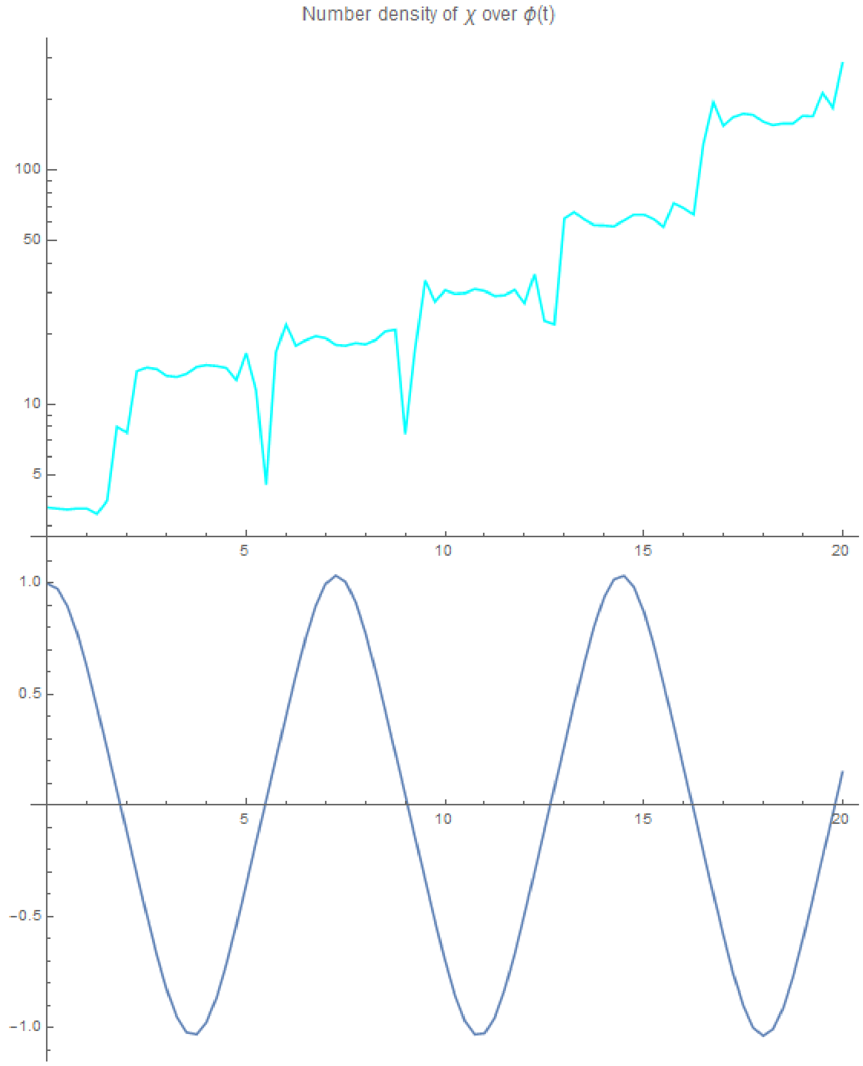

Figure 2: Magnifying the first figure [1] and adding the evolution of the ϕ-oscillations within the same timeframe to show the correlations.

The first figure 1 shows us the evolution of the number density of the χ-field. The timescale is proportinal to the inverted frequency of the ϕ oscillations and we will later see. During preheating we expect an exponential growth of the number density as the growth itself depends on the number of particles already created, which we can clearly confirm in this plot up to a time of 90. Furthermore particles are only allowed to be produced when the inflaton field ϕ stops being adiabatic ($phi(t) = 0$).

The exponential growth of the number density is clearly visible in this figure up to a time of 90. Afterwards the number density of the χ-particle varies, but stays mostly constant, indicating that we left the preheating regime. The changing point around $t = 90$ arises due to the shift from preheating to the elementary reheating. Additionally we can already make out the step-like growth in the first stage of this figure, but the next figure (2) displays this more obvious.

Looking closer at the number density of the χ-particles during preheating in figure 2 the step-like function can be confirmed. To confirm the theory a second function is shown beneath the number density of χ, showing the oscillations of the inflaton field during the same time interval as above. Now each zero crossing of the inflaton field corresponds to the 'jumps' of the step function as we expected and they happen about every $\Delta t \approx \pi$. Only at the zero crossing is the change of the ϕ-field big enough to break the adiabaticity condition. Also the inflaton field has a constant amplitude, showing us that the aforementioned backreaction hasn't started yet.

Expanding the second figure over the whole time interval in figure 3, we can observe the point of transition from preheating to reheating again at around $t \approx 95$. Being in the regime of the elementary theory of reheating the ϕ field is expected to decay until it vanishes in order to get our present universe and at the same time the universe will thermalize.

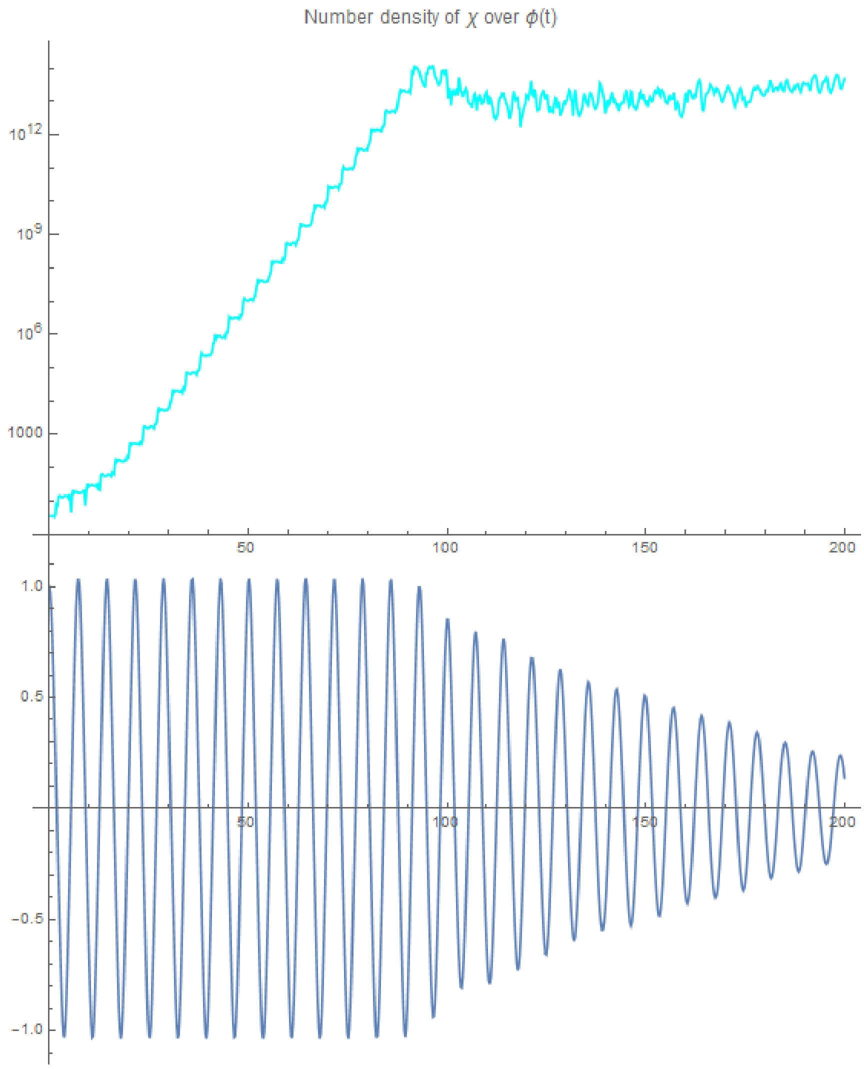

Figure 3: Showing the second figure [2] over a larger timeframe where the transition from preheating to the classical reheating is shown.

The next two figures 4 and 5 display the spectral distributions of the occupation numbers for both, the scalar field χ and the inflaton field ϕ. Again we are looking at the same timeframe as the previous figures, but the different times are now shown as different colored lines. The time starts with the red lines at $t = 0$ and ends with the blue ones at $t = 200$. In each diagram both the axes are scaled logarithmically, thus a vanishing occupation number is not shown anymore in these diagrams.

For the χ-particle occupation number in the upper diagram we start to get a resonant production of particles with $k \approx 3$ and over time over the whole spectrum particles are produced. Without considering backreaction we wouldn't really expect to see inflaton paricles being created anywhere in our theory and looking at the lower figure would be unneccessary. But due to the backreaction we can clearly see a delayed production of inflaton particles, first at $k \approx 3$ and then over the whole spectrum, compared to the production of the scalar field particles. This confirms that this computation takes into account the backreaction according to our theory.

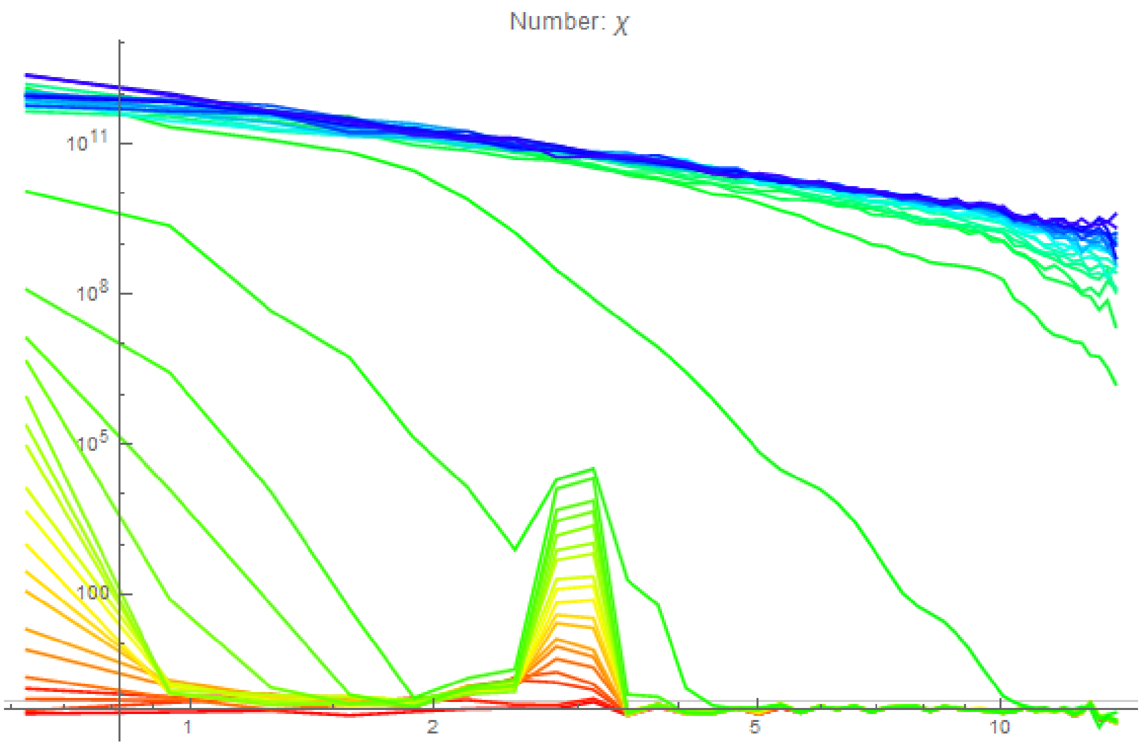

Figure 4: Spectral distribution of the occupation number of the χ-particles. From red to blue the colors represent the time evolution of the occupation numbers.

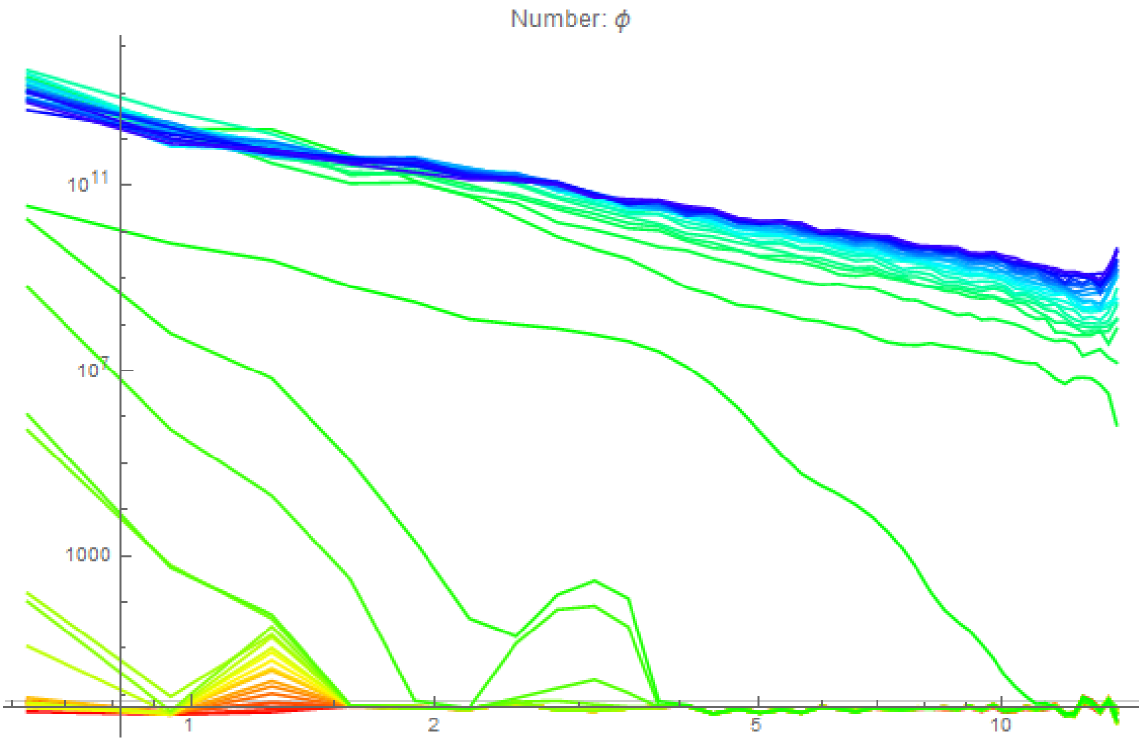

Figure 5: Spectral distribution of the occupation number of the ϕ-particles. From red to blue the colors represent the time evolution of the occupation numbers.

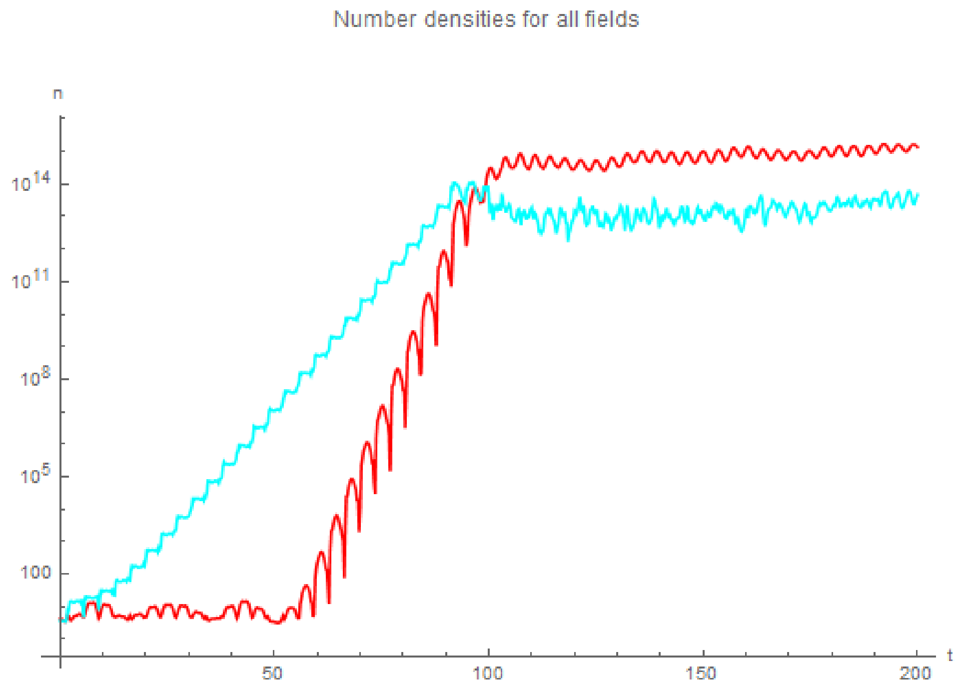

Figure 6: Adding the numberdensity of the ϕ-particles to visualize the effect of the backreaction.

The figure 6 above shows the effects of the backreaction which we already saw in some of the previous figures in a clearer manner. We can now see the starting point of the backreaction which triggers the exponential growth of the ϕ particles until we reach the reheating regime. Because we want the inflaton field and particles to vanish we expect a decline of its number density, which cannot be observed in this timeframe.

For the last diagram 7 the time-evolution of the energy densities for both fields is shown. Like the particle production of the scalar field χ its energy density grows exponentially. Although we have backreaction producing ϕ particles the energy density only decreases by a small fraction. Within the reheating the inflaton field decays and loses energy, which we can barely make out in this diagram.

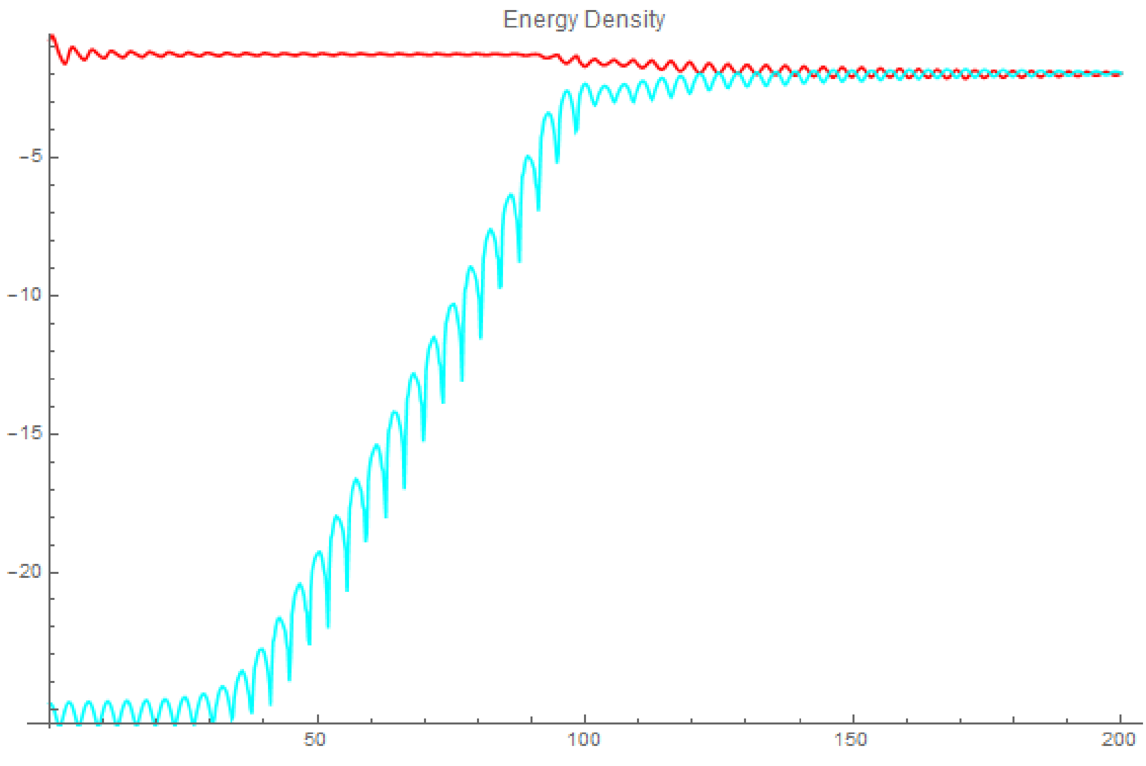

Figure 7: The time-evolution of the energydensities of both fields (χ and ϕ) with logarithmic scaling shows how much energy is transferred from the inflaton-field to the scalar field despite backreaction.

Bibliography

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