Moving Mirrors and Black Hole Radiation[†]

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Abstract: The $1 + 1$ moving mirror model is of particular interest, as it extracts essential thermodynamical features of Hawking radiation. In these notes, I review some of the earlier developments of the model. My approach is to focus on the conceptual rather than the proceeding computations.

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1 Introduction

The 1960's - 80's witnessed a period of rapid development in physics, including new understanding in Black hole physics. Although, it is hard to say who suggested the name 'Black Hole', but 1964 was the first recorded use of the term by a science journalist, Ann Ewing[1], who reported an ealier AAAS meeting, where the term was suggested. As of that time, Einstein's General Relativity (GR) was in full scale - the law of gravity is fully encoded in the Einstein field equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{1}$$

The Einstein's GR also encodes the Copernican Principle, which basically assumes that, there should be no "special" observer in the universe. This *Assumption of Mediocrity* suggests the ordinariness of our place in the universe, that, when applied to the large scale structure of the Universe, the Universe is isotropic and homogeneous. Imposing on the metric to describe a static spherically symmetric spacetime, Schwarzschild solution,

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2 d\Omega^2, \tag{2}$$

became the first modern solution of GR that would characterize a quiescent star.

GR allows for sufficiently massive collapsing object to ultimately undergo continual gravitational collapse under some general conditions. This results in the formation of gravitational singularity, a region where the energy density of the collapsing matter, as well as the spacetime curvature diverge. Indeed, the two singularities in the Schwarzschild metric (2) reveals the true features of a Black hole of mass M. While the singularity at $r = 0$ is an intrinsic singularity, $r = 2M$ is no more real singularity as it is removable by Eddington-Finkelstein coordinate transformation. We now know the nature at $r = 2M$, that everything inside it is trapped, and every trapped null and timelike worldline will inevitably encounter the singularity at $r = 0$. By this, a black hole is characterized by a boundary at $r = 2M$, called the *event horizon*. Object or particle crossing it is doomed, external observers are unaware of events behind the horizon. Black holes are therefore regions of spacetime with strong gravitational influence, that nothing – not even an electromagnetic radiation such as light – can escape from inside it. However, Black holes can also be eternal [2]. They could be thought of as black holes that forever exist, rather than being formed from collapse of some matter. This spherical symmetric solution of the Einstein vacuum field equation involves no matter collapse at all, rather, the event horizon has an unchanging feature of spacetime – it is eternal - thereby leading to some strange consequences such as White holes. Although the geometry of an eternal black hole is identical to that of the vacuum region outside an imploding object, the topology is however different. While it is not known whether there exist in the universe any eternal black hole, or whether all black holes form from collapsing stars, the no-hair theorem essentially requires all black holes be characterized by only three observable classical parameters, the mass (M), electric charge (Q), and angular momentum (J), so that, for most general type of stationary black hole, the area of the horizon is given in plank unit as

$$A = 4\pi\left(r_h^2 + \frac{J^2}{M^2}\right), \tag{3}$$

and it lies at a fixed radial coordinate,

$$r := r_h = M + \sqrt{M^2 - Q^2 - \frac{J^2}{M^2}}. \tag{4}$$

Moreover, developments in quantum theory had risen to an appreciable level of understanding, that a considerable good deal of ideas was known about gravitational collapse up to curvature singularity of a spherically symmetric body, however, generalized understanding of collapse was almost completely unknown. At least, in the late 60s, Kerr solutions[3], had been known to have an electromagnetic and cosological generalizations up to asymptotically flat solutions of the Einstein-Maxwell vacuum field equations. Indeed, Carter[4] had derived equations of motion for charged particle moving in external fields of Kerr-type black holes, using Hamilton-Jacobi theory. This therefore led some to think further about the nature of particles in strong gravitational field.

Following a realization by Beckenstein[5] that black holes should posses a finite, non-zero temperature and entropy, it was clear that there exists a strong correspondence between classical thermodynamics and those of black holes. Classically, black holes have vanishing temperature and, by no–hair theorem, zero entropy. Thus, black hole would be perfectly black, and the laws of black hole thermodynamics would be nothing but a mere analogy. However, Hawking[6] showed from first principles in his fundamental paper, having recognized quantum mechanical uncertainty principle, that black holes are indeed not perfectly black. Indeed, Hawking argument suggests that, pair creation in the gravitational field of a black hole formed by a gravitational collapse leads to black hole evaporation. Black hole would exhibit a steady emission of particles to infinity. The emitted particles have a thermal spectrum corresponding to an effective Hawking temperature

$$T_H = \frac{\hbar\kappa}{2\pi c k_B}, \tag{5}$$

for a Schwarzschild black hole, with surface gravity $\kappa = \frac{c^4}{4GM}$ characterizing the gravitational acceleration experienced at the event horizon, as seen by observer from infinity. Constant k_B is the Boltzmann's constant. In plank unit, $T_H = \frac{\kappa}{2\pi}$.

In what follows immediately is the Fulling-Davies-Unruh effect [7], which predicts the detection of particles in a Minkowski vacuum state by an observer or particle detector at rest in an accelerated Rindler frame. The observer behaves as though it were placed in a thermal bath with temperature $T_U = \frac{a}{2\pi}$, where a is the magnitude of the proper acceleration in Plank unit. The shotcoming of Unruh effect is that, energy required for acceleration by the observer is exponentially large compared with the energy in detected particles, making it practically impossible. However indeed, it turns out that far away from the black hole ($r \gg 2M$), there exists a correspondence between observers in Rindler and Schwarzschild spacetime for limiting identification $a = \frac{1}{4M}$. In this way, accelerated observers at a fixed distance $r \gg 2M$ from black hole detect thermal spectrum of particles with temperature T_H. Density of the observed particle spectrum with energy E for Hawking and Unruh effect therefore takes a similar form,

$$\rho(E) = \left[\exp\left(\frac{E}{T}\right) - 1\right]^{-1}, \tag{6}$$

with κ and a finding correspondence.

In 1976, Davies and Fulling[8] discovered that an accelerating reflecting boundary – mirror – would radiate energy as a result of quantum vacuum disturbance effect. By suitable choice of mirror trajectory, the simple "moving mirror" model provides a very analogous features of black radiation scenarios. Indeed, in the simplicity of the model, it extracts essential thermodynamical features of Hawking radiation. In fact, the mirror trajectory gives meaning to the origin of the black hole geometry. Of important is that, the geometry of this framework is void of curvature unlike Kruskal and Rindler frame in the case of Hawking and Unruh respectively, thus, the quantum fields rather propagate in a flat Minkowski spacetime. Further, it is of note that the moving mirror model generated some controversies between the nature of negative energy effect and black hole thermodynamics -unrestricted energy will cause a serious problem for physics. These made the simple moving mirror model gained some earlier attentions. In this regard, we shall study at a very basic level, some of the earlier developments of this model.

The outline of these notes is as follow. In the next section, we shall build a simple Unruh-DeWitt particle detector. This will provide us a response model to recognise the emergent of particle spectrum. The notion of particle definition is crucial. Therefore, we shall discuss basic idea of cosmological particle creation briefly in section 3. Moving mirror model brought about some important development and discuss in thermodynamics. We shall need to lay key elements of the laws of black hole thermodynaics in section 4. Analysis of simple moving mirror model will follow smoothly in section 5, with more emphasis on conceptual, rather than computational. In section 6, we would like to know how backreaction effect on the mirror could contribute. Usually, this effect is taken as negligible. The remaining sections will focus on the moving mirror thermodynamics. We shall first study the contruction of entropy in section 7 and later conclude in section 8 with a discussion on the issue of possible violation of the second law by negative energy effects.

2 Unruh-DeWitt Particle Detector

In an attempt to extract localized information from quantum fields, such as emergent particle creation in curved spacetime, the idea of particle detector was introduced([7],[9]). In its original, the model is essentially the "particle in a box" detector, simplified such that only two energy levels, say level zero energy state $|0, E_0\rangle$, and exited energy state, $|\psi, E\rangle$, are relevant. Basically, particle detector is a scalar quantum field on its own right, whose excitation is restricted to a cavity such that spatiotemporal information is extracted. In summary, "a particle is what a particle detector detects".

A modified and improved version due to DeWitt, couples a point like particle detector with a field ϕ through its monopole moment μ. Towards contructing an effective particle detector model, one of the questions to ask is the probability of detecting a particle if excitation do occur. This question is answered within the framework of perturbation theory. While there has been approaches up to some higher orders perturbation theory[10], a linear coupling is sufficient for our discuss.

Given a path $x(\tau)$ of an Unruh-DeWitt–type detector through a curved spacetime, the path is parametrized by the detector's proper time τ. Couple the detector with the field ϕ through interaction Hamiltonian,

$$H_{int} = g\mu\phi(x(\tau)), \tag{7}$$

where g is a small coupling constant. For this system, UV divergence arise as a result of the point-like nature of the detector when it is adiabatically switched on or off. Gradual switching on and off can be achieved by involking a switch function $\Lambda(\tau)$ as regulator. $\Lambda = 0$ therefore means the detector is decoupled from the field, and so, there exist no particle production. In this case, the Hamiltonian of the detector field system of finite spatial profile p is given as,

$$H_{int} = g\Lambda(\tau)\mu(\tau)\int_{\mathbb{R}^n} p\left(x(\tau), y\right)\phi(y)dy. \tag{8}$$

The S-matrix element is,

$$S = \langle E, \psi|\, exp\left(-i\int_{-\infty}^{\infty} d\tau H_{int}\right)|0, E_0\rangle \tag{9}$$

through which, up to first order in perturbation theory, we can achieve the transition amplitude matrix,

$$i\mathcal{M} = -ig\,\langle E, \psi| \int_{-\infty}^{\infty} d\tau \Lambda\mu(\tau)\phi(x(\tau))\,|0, E_0\rangle\,. \tag{10}$$

Noting the time evolution $\mu(\tau) = e^{iH_0\tau}\mu(0)e^{-iH_0\tau}$ in interaction picture, with $H_0\,|0, E_0\rangle = E_0\,|0, E_0\rangle$ and $H_0\,|\psi, E\rangle = E\,|\psi, E\rangle$, the transition matrix takes the form,

$$i\mathcal{M} = -ig\,\langle E|\,\mu(0)\,|E_0\rangle \int d\tau e^{i(E-E_0)\tau}\Lambda\,\langle\psi|\,\phi(x)\,|0\rangle\,. \tag{11}$$

The transition probability for the field transiting into an arbitrary excited state $|\psi, E\rangle$ is obtained by squaring the (11). The result is

$$P(E) = g^2\,\langle E_0|\,\mu(0)\,|E\rangle^2\,\mathcal{F}(E - E_0), \tag{12}$$

where we define the particle detector response function which encodes detector trajectory as,

$$\mathcal{F}(E) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{i(E-E_0)\tau}\Lambda^2\mathcal{W}^+(x(\tau), x(\tau')). \tag{13}$$

with

$$\mathcal{W}^+(x(\tau), x(\tau')) := \langle\psi|\,\phi(x(\tau))\phi(x(\tau'))\,|\psi\rangle \tag{14}$$

being the two-point Wightman correlation function. It may be useful to notice that,

i the path of detector and the switch function $\Lambda(\tau)$ are assumed to be sufficiently smooth.

ii for an inertial particle detector, the argument of the resulting δ-function factor in the transition matrix is always greater than 0, hence, transition amplitude vanishes. Indeed, such transition is forbidden by the requirement of energy conservation which must be implemented by the δ-function, an expected consequence of poincare invariance. Transition amplitude is on the other hand non-vanishing for a complicated detector trajectory.

Although, the Unruh-DeWitt detector is reasonably meaningful, however, particle interpretation criterion is not necessarily based on the use of particle detector only. Indeed, thermal bath can also be identified through evaluation of expectation value of energy-momentum tensor accordingly. We should also state that, although, the simple detector model was built for quantized scalar fields, nature indeed allows quantized spinor or vector fields. Infact, some have built detector models for quantized spinor fields for investigating Unruh effect and Hawking radiation of Dirac particles [11]. We note that response function beyond four-dimensional Minkowski spacetime was discussed by Hodgkinson et. al[12]

3 Cosmological Particle Creation

The journey to a consistent second quantization procedure[13] on a curved spacetime was one that came along with difficulty of particle interpretation. The ambiguity of vacuum and its physical interpretation led to a more deeper understanding of black holes. In flat spacetime, the standard procedure is to decompose a field into a combination of a complete set of eigenfunctions, and then quantize it. By extremizing the classical action for a massless hermitian scalar field $\phi(t, x)$,

$$S[\phi] = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi, \tag{15}$$

$\phi(t, x)$ satisfies the Klein-Gordon equation,

$$\eta^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0, \tag{16}$$

whose mode expansion is given as

$$\phi(t, x) := \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2}} \left[a_k \varphi_k^* + a_k^\dagger \varphi_k \right], \tag{17}$$

with $\varphi_k = v_k(t) e^{-ikx}$ being the complete set of positive frequency mode satisfying

$$\begin{cases} (\varphi_i, \varphi_j) = \delta_{ij}, \quad (\varphi_i^*, \varphi_j^*) = -\delta_{ij} \\ \left(\varphi_i, \varphi_j^*\right) = 0. \end{cases} \tag{18}$$

In $2D$, the classical action (15) enjoys infinitesimal conformal symmetry. The metric is invariant under

$$g^{\mu\nu} \to \tilde{g}^{\mu\nu} = \Omega^2(t, x) g^{\mu\nu}. \tag{19}$$

That this transformation is valid only in two dimension is obvious by computation in D-dimension. Indeed,

$$\begin{cases} \sqrt{-g} \longmapsto \left(\Omega^2\right)^{\frac{D}{2}} \sqrt{-g} \\ g^{\mu\nu} \longmapsto \left(\Omega^2\right)^{-1} g^{\mu\nu}, \end{cases} \tag{20}$$

so that, $\int d^Dx \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ is invariant if and only if $D = 2$. Conformal field theories in $2D$ are even more special in that, the group of infinitesimal conformal transformation is infinite, that it allows us to solve the theory exactly and completely. We shall exploit such transformation later, as it will make life easier for us when dealing with the moving mirror problems. Indeed, all physical conclusion in $4D$ is inherited in $2D$ spacetime dimension. However, for now, we work in 4 spacetime dimension (t, x). Readers should note the notation $x = \vec{x}$ in this case for now.

We can read off the mode function of a free field in flat space as $v_k = \frac{1}{\sqrt{w_k}} e^{iw_k t}$, and cannonical commutation relation is,

$$\begin{cases} [\phi(x, t), \phi(y, t)] = 0, \\ [a_k, a_{k'}] = 0, \quad \left[a_k, a_{k'}^\dagger\right] = (2\pi)^3 \delta^3(k - k'). \end{cases} \tag{21}$$

This is the second quantization, to distinguish the quantization of operator ϕ in the Klein–Gordon equation from the old first quantization procedure of one–particle Quantum mechanics, in which ϕ was a wavefunction. Now, for each pair $\left(v_k, a_k^\dagger\right)$, particle definition naturally applies as a global one because of the global properties of the flat Minkowski space. That is, v_k has an invariant meaning on the flat space background, and completely specify the vacuum $|0\rangle$ according as

$$a_k |0\rangle = 0 \qquad \forall \quad k. \tag{22}$$

Howbeit, this particle definition is not carried over into a general curved spacetime. The reason is that, particle definition is done in reference to the underlying geometrical symmetries of spacetime. Indeed, according to wigner's classification[15], particles are nothing but irreducible representation of the poincare group, and so, there will be no killing vectors at all with which to define positive frequency modes. Those symmetries are obviously enjoyed in the Minkowski space, but lost in curved background. In this case, the mode functions lacks invariant meaning and thus, choice of the pair $\left(v_k, a_k^\dagger\right)$, in curved spacetime, does not necessarily gaurantee particle creation. However, it turns out that particle production is realizable as the changing background gravitational field could be expected to create particles, but the expansion of the universe which induce a dynamical gravitational field, leads to an unappreciable particle production[14]. Even a non-dynamical gravitational field, if strong enough, produces particles out of the vacuum.

For either case, vacuum states resulting from a certain isotropic frequency mode φ_i, can have particle definition when considered in pair with another choice of isotropic frequency mode χ_i with an associated creation and annihilation operator $(\tilde{a}_i, \tilde{a}_i^\dagger)$. The different mode functions being related by Bogolyubov transformation

$$\begin{cases} \varphi_j^*(t) := \alpha_{ji} \chi_i(t) + \beta_{ij} \chi_i^*(t), \\ a_j^* := \alpha_{ji} \tilde{a}_i + \beta_{ij} \tilde{a}_i^*. \end{cases} \tag{23}$$

with the Bogolyubov coefficients and the respective completeness relation given as

$$\begin{cases} \alpha_{ij} = (\chi_i, \varphi_j)\,, \quad \beta_{ij} = -\left(\chi_i, \varphi_j^*\right), \\ |\alpha_{ij}|^2 - |\beta_{ij}|^2 = 1. \end{cases} \tag{24}$$

The Bogolyubov transformation (23) shows that the two Fock spaces based on the two choices of modes φ_i and χ_i are different if and only if $\beta_{ij} \neq 0$. In fact, the expectation value of the φ_i-mode particle number operator , $\mathcal{N}_i^{(\varphi)} = a_i^\dagger a_i$, in the χ_i-mode vacuum state $\left|0_{(\chi)}\right\rangle$ is given as

$$\left\langle 0_{(\chi)} \left| \mathcal{N}_i^{(\varphi)} \right| 0_{(\chi)} \right\rangle = \sum_j |\beta_{ij}|^2 . \tag{25}$$

Illustration $\mathcal{I}$: In an asymptotically flat region of spacetime, accordingly, for each positive frequencies (ω_k) mode φ_k with respect to the Minkowski time coordinate, there exists a pair $\left(a_k, a_k^\dagger\right)$. Now, consider in figure 1, a description of a certain spacetime of an initially flat regions I, II, II', followed by a region of curvature III, and then into a final flat region IV. The causal relation between different points in the flat spacetime regions is encoded in Penrose diagram in figure 1b. The metric on the Penrose diagram is conformal to the metric in our flat spacetime according as (19). We shall however not use Penrose diagram in our analysis, as the causal structure is clear enough from figure 1a.

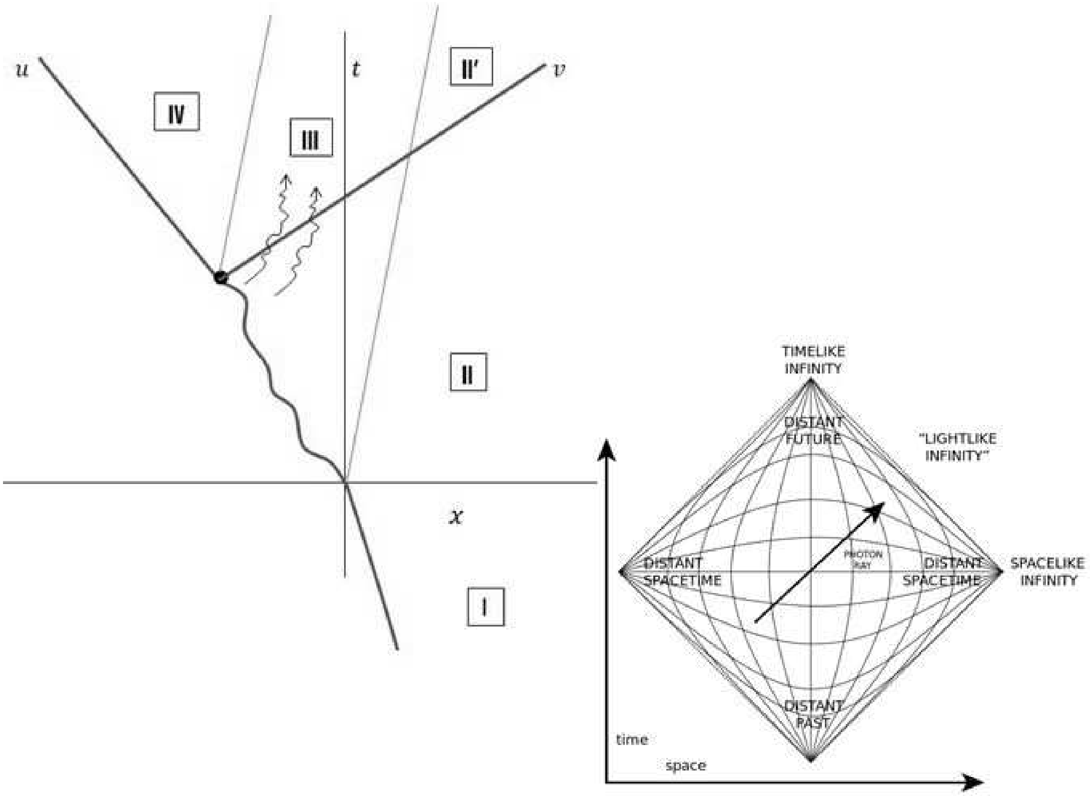

Figure 1: **Left:**(a) Particle production in a region of flat spacetime dominated with curvature; **Right:**(b) Penrose diagram of an infinite Minkowski universe, horizontal axis u, vertical axis v – *Wikipedia.*

The natural complete set of basis is

$$\varphi_k := \begin{cases} \dfrac{1}{\pi\omega_k} \sin(\omega_k x) e^{-i\omega_k t}, & \forall\ k \in I, II, II', \\ \dfrac{1}{\pi\varpi_k} \sin(\varpi_k v) e^{-i\varpi_k u}, & \forall\ k \in IV, \end{cases} \tag{26}$$

with frequency $\omega_k, \varpi_k \in [-\infty, \infty]$. (t, x) and (u, v) are related by an appropriate choice of lorentz transformation. Obviously, positive frequency modes varies from frame to frame, thereby specifying different vacuum and pair $(a_k, a_k^\dagger)$. With this, we can make the following deductions.

i The spacetime region I is Minkowskian and the quantum field resides in the vacuum state $|0_I\rangle$.

All inertial particle detectors register no particles, so that unaccelerated observers identify the quantum state with a physical vacuum $|0_I\rangle$.

ii The spacetime region III is still Minkowskian and fields still live in the quantum state $|0_I\rangle$. However, the initial vacuum $|0_I\rangle$, annihilated according as $a_{k_I}|0_I\rangle = 0$, is no more regarded by inertial observer as the the final vacuum $|0_{III}\rangle$, since $a_{k_{III}}|0_I\rangle \neq 0$. Indeed, this is what is expected in a dynamical gravitational field, causing the creation of particle $|0_I\rangle$ of the scalar field in region III dominated by vacuum definition $|0_{III}\rangle$. Region III, dominated with curvature, can be regarded as a region of "photon production", since in that region, positive and negative frequencies are mixed.

We can further easily effect the propagation of these sets of eigenfunctions into other region of spacetime, however, they are bound to loose their natural form as we shall see for a moving mirror analogy.

4 The Four Laws of Black hole Mechanics

Although the history of black hole thermodynamics began with Bekenstein[5], the prehistory could be traced way back to astronomers studying the conditions for a spherically symmetric, self-gravitating objects in the universe to be in hydrostatic equilibrium. In fact, the classical thermodynamics is well known to be rooted in non-gravitational physics. That its scope finds connection with processes involving black holes scenario is of important relevance to physics. While the fundamental thermodynamic relation,

$$M^2 = \frac{1}{16\pi}A + \frac{4\pi}{A}\left(J^2 + \frac{1}{4}Q^4\right) + \frac{1}{2}Q^2, \qquad (27)$$

featuring all information about the thermodynamical state of black hole matter had been given, a one–to–one correspondence between the laws of classical thermodynamics and those of black hole thermodynamics was later established[16]. It identifies the temperature T with the surface gravity κ, and the horizon area A with Bekenstein-Hawking entropy,

$$S_{BH} = \frac{1}{4}k_B A. \qquad (28)$$

We shall therefore outline the four laws of classical black hole mechanics which encodes the physical properties satisfied by black holes.

The Zeroth Law: The surface gravity κ is constant over the horizon of a stationary black hole. If a proportionality between surface gravity of a black hole and it's temperature is assumed, this law is essentially an analogue of the Zeroth Law of classical thermodynamics which requires the temperature be uniform everywhere in a system in thermal equilibrium.

The First Law: To first order in perturbation of a stationary black hole, the variation in mass in the vacuum case satisfies,

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J + \Phi\delta Q, \qquad (29)$$

where Ω is angular velocity, and Φ is electrostatic potential. These quantities are all defined locally on the horizon, but they are always constant over the horizon of a stationary black hole, just like the surface gravity κ. Analogously, we can identify this law with the first law of classical thermodynamics, whose statement is of energy conservation.

The Second Law: The area A of the event horizon of a black hole is non-decreasing function of time. That is,

$$\frac{\delta A}{\delta t} \geq 0. \qquad (30)$$

Further, two black holes of event horizon areas A_1 and A_2 coalesce to form a black hole with event horizon area A_3, with

$$A_3 > A_1 + A_2. \qquad (31)$$

The second law basically identifies the area of a black hole horizon in correspondence to entropy. However, with matter loss due to Hawking radiation, thereby losing its entropy, this law in its original form violate the second law of thermodynamics. However, a more generalized second law has been established. Basically, the Bekenstein's generalized second law[17] of thermodynamics enforces that, the sum of the combined intrinsic entropy of existing black holes, and the thermodynamical entropy of all matter and radiation fields in the exterior region of the black hole, is non-decreasing. That is,

$$\delta S := \sum_k \delta S_{BH}^{(k)} + \delta S_{matter} + \delta S_{rad} \geq 0. \qquad (32)$$

where $S_{BH}^{(k)}$ is the entropy (28) of the k-th black hole. Later with a moving mirror model, we shall look more closely on an interesting issue regarding possible violation of the second law by negative energy effect.

The Third Law: The surface gravity κ cannot be lowered to the absolute zero in any finite number of operations by any procedure, regardless of how well or idealized the procedure is constructed. That κ cannot go to zero is analogous to the third law of classical thermodynamics which requires that, the entropy of a system at absolute zero is a well-defined constant.

5 1+1 Moving Mirror Model

As motivated in the previous sections, particle production is possible in region of spacetime, dominated with curvature e.g. Unruh-Hawking particle production. It is however desireable to seek for a mechanism through which particle production is possible without dealing with geometries with high curvatures such as the Rindler and Kruskal frame. If we think about the very flat Minkowski space, then, an immediate question is, what could be the simplest situation for a nontrivial modification of the minkowski vacuum $|0_M\rangle$? An interesting approach to reach out for such modification is the consideration of a flat spacetime equipped with a sufficiently reflecting barrier or mirror. Indeed, Davies and Fulling's [8] simple $1+1$ dimensional moving mirror model in flat Minkowski space, provides a reminiscent of the black hole scenario. Under reasonable conditions, such as suitable choice of mirror trajectory, moving mirrors emulate most features of the black hole radiation. We shall now sketch a field-theoretic treatment of this model.

In $2D$, a barrier or moving mirror degenerates into a single reflecting point in space, moving along a trajectory, it's worldline. In general, a moving mirror worldline follows a trajectory,

$$x = z(t), \quad |\dot{z}(t)| < 1. \tag{33}$$

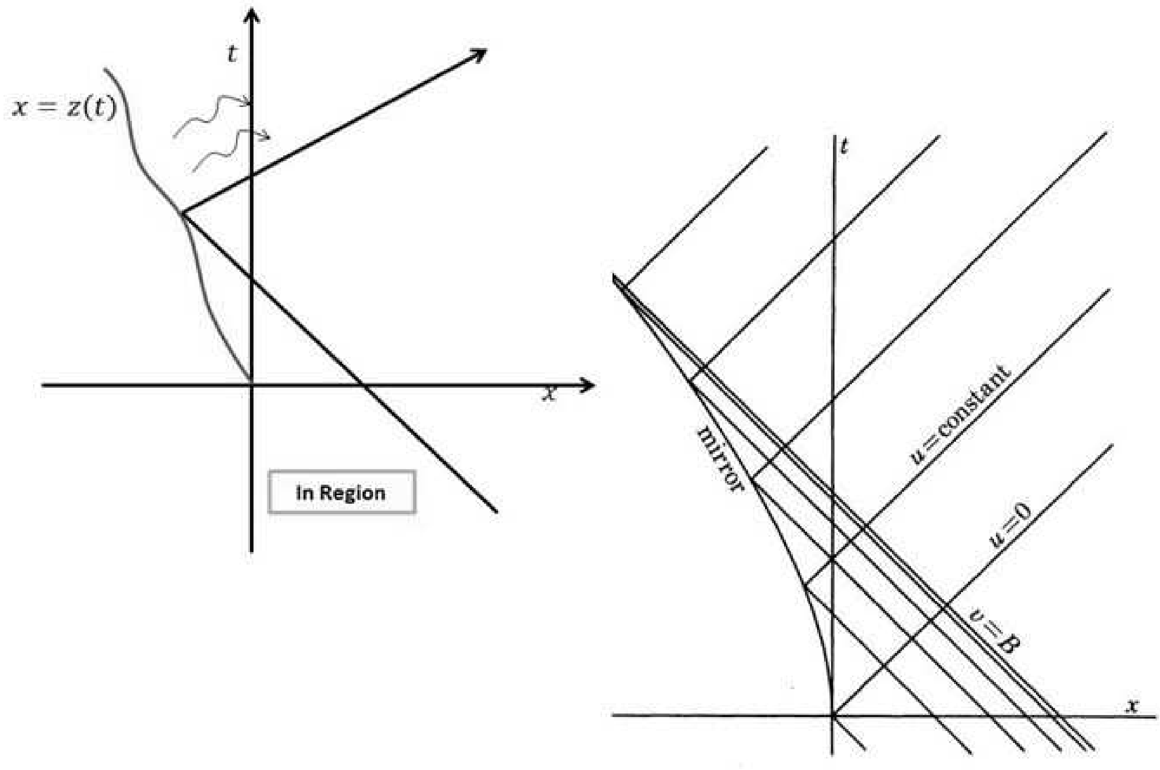

Figure 2: **Left:**(a) Particle production by moving mirror, with vanishing scalar field ϕ at the boundary $x = z(t)$. Incoming Null rays reflected to the right of the mirror. Mirror motion at $t > 0$ induces doppler shift suffered by incoming wave mode; **Right:**(b) Particle production in the late time asymptote of a moving mirror traectory. – *Davies and Fulling, '77*

We shall particularly consider a mirror trajectory which join smoothly onto a static trajectory $z(t)=0$ for region $t<0$, and later consider the mirror trajectory at a very late time asymptote, $t\to\infty$, a region relevant for particle production. The very early time behaviour of the mirror is irrelevant in particle production. We shall quantize a massless scalar field $\phi(x,t)$, restricted to the right of the mirror. Basically, we seek to quantize the scalar field, satisfying,

$$\begin{cases} \dfrac{\partial\phi}{\partial t}-\dfrac{\partial\phi}{\partial x}=0, & x\geq z(t),\\ \phi(t,z(t))=0. \end{cases} \tag{34}$$

The wave equation is nothing but the Klein-Gordon equation (16) in $2D$, satisfying a reflecting boundary condition. All physics in four dimension are inherited into two spacetime dimension. For our analysis, we wish to construct null coordinate system (u,v), or say a lightcone coordinate, which coincides with (t,x) throughout the whole region $t\leq x$.
This problem is most exploited by conformal invariance enjoyed by the classical action. Indeed, under a conformal transformation

$$\begin{cases} t-x=f(u-v),\\ t-x=g(u+v), \end{cases} \tag{35}$$

the minkowski metric remain flat up to an overall rescaling, i.e.

$$dt^2-dx^2=f'(u-v)g'(u+v)(du^2-dv^2). \tag{36}$$

It is possible to choose f and g such that, $v=0$ coincides with the mirror trajectory $x=z(t)$, so that problem (34) becomes,

$$\begin{cases} \dfrac{\partial\phi}{\partial u}-\dfrac{\partial\phi}{\partial v}=0,\\ \phi(u,0)=0. \end{cases} \tag{37}$$

The exploitation of conformal symmetry implies that, minimally coupled massless scalar field in the 1+1 Minkowski spacetime dimension is in fact conformally coupled. Also, the mirror trajectory becomes

$$\frac{1}{2}\left[g(u)-f(u)\right]=z\left(\frac{1}{2}\left[g(u)+f(u)\right]\right), \tag{38}$$

Solutions of (38) exist globally for mirror motions. We shall have an illustration where we consider such solution at the very late time. The wave equation (37) has a generalized positive mode solution

$$\varphi_\omega=F_\omega\left(v-u\right)+G_\omega\left(v+u\right), \quad \omega>0. \tag{39}$$

G_ω is completely determined at the in-region with a general basis,

$$G_\omega(v)=-\frac{1}{i2\sqrt{\pi\omega}}e^{-i\omega v}. \tag{40}$$

F_ω has a similar form only for positive argument. To obtain the general form, we subect φ_ω to the boundary condition at a certain point $(\tau_u, z(\tau_u))$ on the mirror. We would have

$$0=\varphi_\omega=F_\omega\left(z(\tau_u)-\tau_u\right)+G_\omega\left(z(\tau_u)+\tau_u\right)$$

. leading to

$$F_\omega\left(z(\tau_u)-\tau_u\right)=\frac{1}{i2\sqrt{\pi\omega}}e^{-i\omega(z(\tau_u)+\tau_u)}. \tag{41}$$

By recognising from the conformal transformation up to rescaling that, τ_u is determined implicitly by trajectory through

$$\tau_u-u:=x=:z(\tau_u), \tag{42}$$

we can finally bring scalar field ϕ into a form

$$\varphi_\omega=\frac{i}{\sqrt{4\pi\omega}}\left(e^{-i\omega v}-e^{-i\omega(2\tau_u-u)}\right). \tag{43}$$

Let time τ_u be related by definition

$$p(u)=2\tau_u-u. \tag{44}$$

We can realize that,

$$p(u):=\tau_u+\tau_u-u=\tau_u+z(\tau_u)=:v.$$

Cconversely, we can achieve

$$v=p(u)\Leftrightarrow u=f(v) \quad with \;\; p=f^{-1}. \tag{45}$$

This results in the realization of two different representations of mode φ_ω solutions,

$$\begin{cases} \varphi^{in}_\omega=\dfrac{1}{i2\sqrt{\pi\omega}}\left(e^{-i\omega v}-e^{-i\omega p(u)}\right),\\ \varphi^{out}_{\omega'}=\dfrac{1}{i2\sqrt{\pi\omega'}}\left(e^{-i\omega' f(v)}-e^{-i\omega' u}\right), \end{cases} \tag{46}$$

where ω and ω' are related by doppler factor δ. The two representations are related by Bogolyubov transformation (23). This realization has an immediate meaning. Functions f and p have a special feature in the following sense. An incident plane wave, $e^{-i\omega v}$, is reflected by the moving mirror to yield an outgoing wave of the form $e^{-i\omega p(u)}$, and conversely, an outgoing wave $e^{-i\omega' u}$ from the right of the mirror, when traced backward in time to the mirror becomes $e^{-i\omega' f(v)}$. The changing of the standard form of the incident modes into the conplicated form $e^{-i\omega p(u)}$ is as a result of a the doppler shift suffered by the incident modes at the instant of reflection from the moving mirror. Indeed, by noticing that the function $2\tau_u - u$ is unchanged all along the null ray $u = constant$ from the mirror surface to the right of the mirror, we can gain an immediate intuition that the distortion of the modes from its standard form occurs very suddenly upon reflection. What does this means physically? To find out, we shall need to analyse the particle interpretation using Unruh-DeWitt particle detector.

First, the solution to (34) for the fully contrained scalar field ϕ by the boundary of the moving mirror are

$$\phi(t,x) = \begin{cases} \int_0^\infty d\omega \left[a_\omega \varphi_\omega^{in} + a_\omega^\dagger (\varphi_\omega^{in})^* \right], \\ \int_0^\infty d\omega \left[\tilde{a}_\omega \varphi_\omega^{out} + \tilde{a}_\omega^\dagger (\varphi_\omega^{out})^* \right]. \end{cases} \tag{47}$$

with $d\omega = \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2}}$. Either of the solutions is applicable depending on consideration of the incident null ray. The 'in' solution is particularly for a null ray reflected to the right of the mirror, while the 'out' solution is the converse of the 'in' solution. Second quantization follows according as (21).

Particle Interpretation

We begin by recognising operators a_ω and $a_\omega^\dagger$ as annihilation and creation operators for which quanta are the familiar particles of the standard QFT, i.e. following (22). However, particle interpretation is restricted by our ealier definition of particles, which requires that, " a particle is what a particle detector detects." Such definition encodes a precision observer. In this case, we would need to analyse such interpretation at two regions. First at region $t \leq 0$, a region where mirror is at rest or static, and then at $t > 0$, a region where the mirror undergoes a period of acceleration. In this two regions, we ask whether any pair $(\varphi_\omega, a_\omega)$ gives particle interpretation.

Region $t \leq 0$

Pick a vacuum completely specified by φ_ω^{in}, say $|0_{in}\rangle$. We then ask:

Does $|0_{in}\rangle$ holds a particle meaning?

In this region, $t \leq 0$, the mirror is static, $z(t) = 0$. By (42), $\tau_u - u = 0$, and modes solution (43) reduce to

$$\varphi_\omega^{in} = \frac{i}{\sqrt{4\pi\omega}} \left(e^{-i\omega v} - e^{-i\omega u} \right), \tag{48}$$

from which $\phi(t,x)$ can be deduced. Following Birrel[18], the computation of Wightman function (14) yields

$$\mathcal{W}^+(x(\tau), x(\tau')) = \langle 0_{in}| \phi(x(\tau))\phi(x(\tau')) |0_{in}\rangle$$

$$\mathcal{W}^+ = -\frac{1}{4\pi} \ln \left[\frac{(u-u'-i\epsilon)(v-v'-i\epsilon)}{(v-u'-i\epsilon)(u-v'-i\epsilon)} \right] \tag{49}$$

By subtituting of (49) in the response function (13) for computation of transition probability (12), one can verify that an inertial particle detector with trajectory switched off outside the "in" region records no particle. Indeed, associated transition amplitude vanishes, with transition forbidden on energy conservation groud. The presence of the mirror does not excite the detector at the region $t \leq 0$, hence, no particle production.

Region $t > 0$

In this region, we shall ask same question probed at the region $t \leq 0$. Key difference is that, the mirror undergoes a period of acceleration. Since the mirror is not at rest, $z(t) \neq 0$, the field mode solution is the full form (46), and we can therefore deduce $\phi(t,x)$ accordingly. Computing the Wightman function for this region,

$$\mathcal{W}^+ = -\frac{1}{4\pi} \ln \left[\frac{(p(u)-p(u')-i\epsilon)(v-v'-i\epsilon)}{(v-p(u')-i\epsilon)(p(u)-v'-i\epsilon)} \right] \tag{50}$$

where $p(u)$ is define in (44). We can verify that, on subtitution of the Wightman function (50) into (13), a particle detector in general trajectory predicts a non-zero response. This result is anticipated. Indeed, we earlier gained an immediate intuition that the doppler distortion of the standard null ray modes $e^{-i\omega v}$ into the form $e^{-i\omega(2\tau_u - u)}$ occured very suddenly upon refection. In fact, tracing $e^{-i\omega(2\tau_u - u)}$ backward in time reduces it to the standard form $e^{-i\omega u}$ associated with $|0_{in}\rangle$, only at $u < 0$, but not at the region $u > 0$. We can at least, for this reason, understand that the associated $|0_{in}\rangle$ is no more a physical vacuum state at region $u > 0$ corresponding to a region $t > 0$ by conformal map. That's the doppler shift experienced on the surface of the accelerated mirror excites the field modes to cause particle creation. By specifying a certain example of a trajectory (33), we shall now demonstrate that this indeed leads to a thermal flux of radiation.

Illustration $\mathcal{II}$: Consider a perfect mirror starting from rest and accelerating for an infnite time along the trajectory. Somewhere along it's motion, it reverts to a uniform acceleration. One particular asymptotic trajectory of interest, which smoothly joins onto a static configuration at time $t \leq 0$ is

$$x := z(t) = -\frac{1}{\kappa}\ln(\cosh\kappa t), \quad \kappa > 0, \tag{51}$$

where the e-folding time κ parametrizes the trajectory of the accelerating mirror. As we have seen ealier, the behaviour of the mirror and hence, the mirror trajectory at the very early time will be irrelevant for our discuss. We shall therefore consider only the asymtotic regime at the very late time, $t \to \infty$. Late time asymptotic behaviour of (51) takes a general form,

$$x := z(t) \;\; -t - Ae^{-2\kappa t} + B, \quad as \quad t \to \infty, \tag{52}$$

with $A, B, \kappa \in \mathbb{R}^+$. The case $z(t) = -\ln\cosh t$ corresponds to $A = 1$, $\kappa = 1$, and $B = ln2$. This trajectory is fully represented in Figure 2b. Towards the left, the mirror recedes rapidly with an increasing acceleration, reaching out to the speed of light asymptotically. For the case $A = \frac{1}{\kappa}$, the velocity is

$$V(t) := \frac{dz(t)}{dt} \;\; -1 - 2e^{-2\kappa t}, \tag{53}$$

We can also deduce function f in relation (45) for such late outgoing null ray. A very late outgoing null ray along $u = \tilde{u}$ is reflected at late time mirror traectory point, say $(\tilde{u}, \tilde{B})$, at time dictatated by relations (44) and (45).

$$\tilde{B} := p(\tilde{u}) = 2t - \tilde{u}$$

The time taken by the ray is

$$t = \frac{1}{2}\left(\tilde{u} + \tilde{B}\right). \tag{54}$$

Subtituting (54) in (52), and noticing $z(t) + t = \tilde{B}$, we can verify that

$$\tilde{u} := -B - \frac{1}{\kappa}\ln\left[-\kappa(\tilde{B} - B)\right] = f(\tilde{B}). \tag{55}$$

As obvious from Figure 2b, $v = B$ features a horizon, after which incoming null rays travel undisturbed, that is, without encountering the mirror, and hence not reflected. Indeed, the mirror trajectory represents the origin of black hole geometry.

Since we have earlier completely solved this system and quantized it, and further understand the 'in' and 'out' mode representations (46) are related by Bogolyubov transformation (23), we can proceed to deduce the mean number particles (25) of the radiation flux at the very late time. The Bogolyubov coefficient are overlaps between the incoming waves from the 'in' and 'out' mode representations. Noticing that $p = f^{-1}$, one obtains up to normalization,

$$\begin{cases} \alpha_{\omega\omega'} = \left(\varphi^{out}_{\omega}, \varphi^{in}_{\omega'}\right) = \left(e^{-\omega v}, e^{-i\omega' f(v)}\right), \\ \beta_{\omega\omega'} = -\left(\varphi^{out}_{\omega}, \varphi^{in*}_{\omega'}\right) = -\left(e^{-\omega v}, e^{i\omega' f(v)}\right), \end{cases} \tag{56}$$

On integrating by parts,

$$\begin{aligned} \left\{ \begin{array}{c} \alpha_{\omega\omega'} \\ \beta_{\omega\omega'} \end{array} \right\} &= \pm\frac{i}{2\pi}\sqrt{\frac{\omega}{\omega'}}\int_0^B e^{\pm i\omega' f(v) - i\omega v} dv \\ &= \mp\frac{1}{2\pi\sqrt{\omega\omega'}} e^{iB(\omega'-\omega)} \frac{i\kappa}{\omega}^{\mp i\omega'/\kappa} \int_0^{-i\omega' B} x^{\mp i\omega'/\kappa} e^{-x} dx. \end{aligned}$$

Recognizing the Gamma function $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}$,

$$\left\{ \begin{array}{c} \alpha_{\omega\omega'} \\ \beta_{\omega\omega'} \end{array} \right\} = \mp\frac{1}{\sqrt{4\pi^2\omega\omega'}} e^{\pm\pi\omega'/2\kappa} e^{iB(\omega'-\omega)} \left(\frac{\omega}{\kappa}\right)^{\pm i\omega'/\kappa} \Gamma\left(1 \mp \frac{i\omega}{\kappa}\right) \tag{57}$$

From this, the outgoing radiation spectrum is given by

$$\Xi(\omega') = \left\langle 0_{in} \left| \mathcal{N}^{(out)}_{\omega'} \right| 0_{in} \right\rangle = \int_0^\infty d\omega \, |\beta_{\omega\omega'}|^2 . \quad (58)$$

We can use completeness relation (24) to bring (58) into

$$\Xi(\omega') = \int_0^\infty d\omega \frac{1}{\left|\frac{\alpha_{\omega\omega'}}{\beta_{\omega\omega'}}\right|^2 - 1}, \quad (59)$$

from which we can verify that

$$\Xi(\omega') = \frac{1}{e^{\omega'/k_B T} - 1}. \quad (60)$$

with

$$T = \frac{\kappa}{2\pi k_B}. \quad (61)$$

Indeed, $\Xi(\omega')$ is essentially the distribution one would expect for a bosonic thermal radiation at a temperature T, a result that shows that the moving mirror emulates precisely as a black body. In fact, this is an exact same spectrum(5) computed by Hawking [6] for a Black hole which exhibits a steady emission of particles to infnity.

6 Backreaction on Moving Mirror

Given a system, nature can allow for possibility of some sort of external objects to have a measurable influence on the overall system. On a large scale, we can understand backreaction as the influence of density inhomogeneities on average properties of the Universe. However, when the system behaves quantum mechanically, backreaction is usually not taken into account as its effect is considered to be very small. In this case, backreaction is negleted and particles therefore move along a geodesic of the unperturbed spacetime. Similar issue was the case of the Hawking semiclassical effect [6], where backreaction effect of the quantum fluctuations on the metric was considered negligible. This has led to the very much debate on black hole information problem. Also, in the simple moving mirror model of Davies and Fulling [8], the backreaction was ignored. Therefore, it is interesting to understand the effect of backreaction on moving mirror, and for this, we shall consider the effect of some external force, acting on the mirror particle.

The first key insight is a modification of the simple $1+1$-dimensional moving mirror model, wherein Kentaro et. al.[19] proposed a Lagrangian density,

$$\mathcal{L} = \frac{1}{2}\int_{z(t)}^\infty dx \partial_\mu\phi\partial^\mu\phi - m\sqrt{1-\dot{z}^2} + \xi z \quad (62)$$

of a closed system. The Lagrangian features real massless scalar field confined by a moving mirror, pushed by a constant force. By extremizing the action, the dynamic of the system is given by equations of motion

$$\begin{cases} \dfrac{\partial\phi}{\partial t} - \dfrac{\partial\phi}{\partial x} = 0, & x \geq z(t), \\ \dot{z}(t)\dfrac{\partial\phi}{\partial t} + \dfrac{\partial\phi}{\partial x} = 0, & x = z(t), \\ \dfrac{d}{dt}\left(\dfrac{m\dot{z}}{\sqrt{1-\dot{z}^2}}\right) = \xi - \dfrac{1}{2}\partial_\mu\phi\partial^\mu\phi. \end{cases} \quad (63)$$

This dynamics is a reminiscent of equation (34) of the moving mirror problem, that is, (63) features field equation, boundary condition and equation of motion of the mirror. Therefore, when analysed using a method similar to our earlier treatment, it yields a planckian distribution with a temperature given by (61). The equation of motion of the mirror is immediately useful for the purpose of analysing the effect of backreaction on the mirror. In this equation, replace the field ϕ by its VeV,

$$m\frac{d}{dt}\left(\frac{\dot{z}}{\sqrt{1-\dot{z}^2}}\right) = \xi - \frac{1}{2}\langle\partial_\mu\phi\partial^\mu\phi\rangle,$$

and then solve for the VeV. It is possible to use an approach similar to those used in [8] to show that, for an arbitrary mirror motion,

$$\langle\partial_\mu\phi\partial^\mu\phi\rangle = -\frac{1}{12\pi}\left[\frac{2\dddot{z}}{(1-\dot{z}^2)^2} + (5+6\dot{z})\frac{\ddot{z}^2}{(1-\dot{z}^2)^3}\right],$$

where the irrelevant divergent part has been discarded. This divergent part appears at the region where the mirror is at rest. This procedure therefore leads to an equation of mirror motion

$$\frac{d}{dt}\left(\frac{m\dot{z}}{\sqrt{1-\dot{z}^2}}\right) = \xi + \frac{\dddot{z}}{2\pi(1-\dot{z}^2)^2} + \frac{\ddot{z}^2(5+6\dot{z})}{24\pi(1-\dot{z}^2)^3}. \quad (64)$$

Define the proper acceleration

$$a = \frac{dP}{dt} \quad with \quad P = \frac{\dot{z}}{\sqrt{1-\dot{z}^2}},$$

we can verify that

$$\dot{a} = \frac{12\pi}{\sqrt{1+p^2}}\left(-\frac{5}{24\pi}a^2 + ma - \xi\right). \quad (65)$$

Therefore, as expected by the nature of the mirror (53), $\dot{a}$ is always negative, and therefore, acceleration decreases perhaps to a less degree, as the external force effected by ξ on the mirror becomes smaller.

With this, it is of interest to find out the effect of backreaction on the incoming and outgoing null rays. Afterall, by negleting the backreaction effect on the mirror for trajectory of the generalized form (52), and then use energy conservation $p+k = p'+k'$ at the point of reflection on the moving mirror, the resulting Doppler relation

$$\omega' = \left(\delta^2 - \frac{2\omega}{m}\delta\right)^{-1} \omega, \qquad (66)$$

shows that, the incoming frequency modes ω is shifted quadratically into the outgoing frequency modes ω' for a massless scalar field. $\delta = \sqrt{\frac{1-V}{1+V}}$ is the Doppler factor. This will then mean that, the energy of the outgoing null ray will go large, and energy conservation is thereby violated. However, a computation by [20], based on the use of a combined WKB and saddle-point approximation to implement energy conservation, showed that, by taking taking quantum mechanical backreaction effect into consideration, the Doppler relation should take a form,

$$\omega' \approx \left[\kappa(B-v) + \frac{2\omega'}{m}\sqrt{\kappa(B-v)}\right]\omega. \qquad (67)$$

with a doppler factor $\delta = \frac{1}{\sqrt{\kappa(B-v)}}$. This result is at least reasonable. For one, one could notice that, for a finite mirror mass, the incoming energy always remains smaller than the kinetic energy of the mirror just after the collision. Indeed, ω cannot be infinitely large, instead, it should grow linearly with the doppler factor δ. Infact, [20] further showed that, backreaction effects lead to temperature (61) being halved, i.e, $T = \frac{\kappa}{4\pi k_B}$.

7 Moving Mirror Entropy

We introduced in the last section that, backreaction effect leads to information problem, one of the puzzles arising from Hawking radiation. Hawking argued in his fundamental paper [6] that, a black hole, formed out of a pure quantum state, evolves to a totally uncorrelated thermal mixed state, thereby violate the law of quantum mechanics, since this is impossible in a unitarily evolving system. This led to some controversial debates on black hole information paradox. However, D-brane interpretation has led to some suggestion of the existence of a unitary S-matrix description for the Hawking radiation. In this interpretation, the Bekenstein-Hawking Entropy, S_{BH}, of a near extremal black hole is the logarithm of the number of different states of open strings ending on the D-brane [21]. Therefore, amount of inaccessible information is related with entropy. This provide some motivations to study whether it is possible to assign entropy to a manifestly unitary process. An example of such unitary process is the moving mirror effect.

Motivated by this, Mukohyama and Israel [22] proposed a definition of moving-mirror entropy associated with temporarily inaccessible information. They took a key insight from von Neumann conditional entropy, a generalized definition proposed by Cerf et.al [23], using correspondence between classical conditional entropy and quantum von Neumann entropy. In this definition, the conditional entropy of a subsystem 1 relative to 2 is given as

$$S(1|2) = -\mathbf{Tr}\left[\rho_{12}\ln\rho_{1|2}\right], \qquad (68)$$

where

$$\rho_{1|2} = exp(-\sigma_{12}), \quad with \quad \sigma_{12} = \mathbf{1}_1 \otimes \ln\rho_2 - \ln\rho_{12},$$

is the conditional “amplitude” density matrix, representing a quantum generalization of the conditional probability. Also, $\mathbf{1}_1$ is the unit matrix in the Hilbert space of subsystem 1. For the moving mirror, the Fock spaces constructed from Hilbert spaces of the subsystems are symmetric, and so, the density matrix ρ_{12} represents a pure state. For pure state ρ_{12},

$$S(1|2) = -S_{ent} \quad with \quad S_{ent} = -\mathbf{Tr}\left[\rho_2\ln\rho_2\right]. \qquad (69)$$

S_{ent} is called entanglement entropy which can be calculated separately for the right(R) and left(L) sectors separated by the mirror. The total entanglement entropy is the sum

$$S_{ent} = S_{entR} + S_{entL}. \qquad (70)$$

For a mirror trajectory $x^\mu(\tau)$, parametrized by proper time τ along the mirror trajectory, we know it is possible to obatain the trajectory relation $v = p((u)$ (45). By introducing a new coordinate $\tilde{u}$ according as $\tilde{u} = p(u)$ and introducing infrared $L_{\tilde{u}}$ and ultraviolet $l_{\tilde{u}}$ cutoffs in the $\tilde{u}$-coordinate, [22] computed the total entaglement entropy as

$$S_{ent} = -\frac{1}{144}l^2a^2\left[1 + \mathcal{O}(la, l\partial_\tau \ln a)\right] - \frac{1}{6}\ln\left(\frac{L}{l}\right)$$

where $a^2 = \frac{d^2x^\mu}{d\tau^2}\frac{d^2x_\mu}{d\tau^2}$ is the proper acceleration and L and l are respectively infrared and ultraviolet cutoffs in the proper time τ given by the relations

$$\begin{cases} l_{\tilde{u}}l_{\tilde{v}} = l^2\left[1+\frac{1}{12}l^2a^2+\mathcal{O}(l^3a^3, l^3a\partial_\tau a)\right], \\ L_{\tilde{u}}L_{\tilde{v}} = L^2. \end{cases} \tag{71}$$

By subtracting $S(1|2)$ for a non-accelerating mirror from those of actual trajectory, define the moving mirror entropy is defined as

$$S_{MM}(\tau_0) := S(1|2) - S(1|2)\,|_{a=0},$$

which one can verify to give

$$S_{MM}(\tau_0) := \frac{1}{144}l^2a^2(\tau_0)\left[1+\mathcal{O}(la, l\partial_\tau \ln a)\right], \tag{72}$$

so that it measures how much the motion of the mirror increases the uncertainty of the quantum state of wave packet modes which are reflected by the mirror after τ_0.

8 Moving Mirror & the 2nd Law

At classical level, local energy densities for most physical fields are known to be positive definite. The energy-momentum density, as seen by an observer with a timelike, future-directed 4-velocity v_μ, satisfy the dominant energy condition,

$$T_{\mu\nu}v^\mu v^\nu \geq 0. \tag{73}$$

This is not necessarily true at the quantum level, where suppression of vacuum fluctuations is allowed, thereby leading to the emergence of sub-vacuum phenomena. An example of sub-vacuum phenomena is negative energy density. In this case, vacuum flunctuation is effectively suppressed. Indeed, it was shown [24] that, it is possible to contruct quantum states in which renormalized expectation value of local energy density at a given point is arbitrarily negative. One of such contruction features in moving mirror radiation, where flux of energy from the surface of the mirror can be negative.

Given this situation, Ford[25] argued that the existence of negative energy fluxes would lead to a breakdown of the second law of black hole thermodynamics (32). Basically, this mean that absorption of negative energy flux F by a hot body, like black hole, would decrease the body's temperature, and consequently, its entropy, however, except only if the negative energy flux is contrained by an uncertainty limit

$$|F|\,\tau^2 \lesssim 1. \tag{74}$$

In this regard, negative energy flux must be prolonged over a time scale τ for there to be reduction in entropy, with entropy decreasing microscopically according as

$$|S| \lesssim 8\pi k_B, \tag{75}$$

making the process statistically irrelevant.

Motivated by this, Davies [26] re-examined the moving mirror problem under Ford's limit (74) and (75), and concluded with a suggestion that, the second law is violated if the negative energy flux F is slowly varying. This would mean that, under some certain conditions, moving mirrors can violate the second law. This led to few arguments and questions [27]. Of interest is a response by Deutsch *et al.* They emphatically argued against the suggestion that, processes involving hot body absorption of negative energy would violate the second law, and also falsify the claim that, sufficiently small fall in entropy can be regarded as a statistical fluctuation that it could be ignored. Their argument was based on two possible descriptions of negative energy effect, in relation to entropy of an uncorrelated body undergoing a mechanism similar to moving mirror scenario. Their argument can be summarized as follows.

i Epstein *et al*[24] negative energy effect can be understood either as a static vacuum polarisation effect or a coherence effect between different states of quantum field.

ii Static vacuum polarisation effect description is appropriate for quasistatic systems such as Casimir effect or vacuum energy of a closed universe. Coherence effect description is appropriate for non-quasistatic, e.g energy flux from moving mirrors.

iii **Quasistatic system:** Since entropy (68) is independent of the labelling of the states, and thermodynamic beta, $\beta = k_B^{-1}\partial S/\partial E$, is independent of zero of energy, then lowering zero of energy without changing the occupancy of states will not affect the entropy or temperature of the body.

iv **Non-quasistatic system:** Effect of the coherence flux of radiation on an uncorrelated body is a sum of effects of its component parts. Since all component particles of the flux have positive energy, they slow down the rate at which the hot body cools, an effect not qualitatively different from effect of positive energy flux.

v In the two systems, negative energy densities or fluxes will not be absorbed by hot uncorrelated bodies, and therefore, entropy is always nondecreasing.

Among other very earlier investigation is by Grove [27], who have shown that negative energy fluxes from moving mirrors may be absorbed by a particle detector, but violation of the second law by such fluxes is unlikely. While it was not so very clear whether or not the second law is broken by negative energy fluxes, there have been suspicions that the laws of physics do place restrictions on negative energy effects. Ford *et al*[28] has therefore provided some "quantum inequalities" which constrain F and τ to sufficiently prevent quantum coherence effects from producing such large scale effects as gross violations of the second law of thermodynamics or of cosmic censorship. Indeed, unrestricted negative energy would cause serious problems for physics.

9 Discussion

We have seen a moving mirror model which extracts main thermodynamical features of Hawking radiation. While we retrict the discussion to a single mirror model, two-mirror problem has been treated in [8], and similarly [29]. Some classes of mirror trajectories, $z(t)$, have also been considered in few literatures. Constant velocity trajectory, Proex and Carlitz-Willey are among few other trajectories already treated. Good *et al*[30] has few of the list.

After the issuance of "quantum inequalities" contrainsts on negative energy fluxes and density, there may have been a further proof of the existence of negative energy density in quantum field theory. Based on this, some and including Davies [31], have further worked on particle detector model for detection of negative energy, and there has been some calls and proposal for a realistic experiment [32].

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