RELATIVITY AND COSMOLOGY I

Problem Set 3b 11 October 2013

1. Gravitational redshift

Consider a typical neutron star of 1.4 solar masses and 10 km radius emitting light with the wavelength associated to the Lyman-α line of the hydrogen atom. Which would be the wavelength observed in Earth ? Assume that the relative velocity between the neutron star and the Earth is completely negligible. Which are the implications of this assumptions ?
Indication: The Lyman-α line corresponds to the transition $2p^{1/2} \to 1s^{1/2}$.
$G = 6.673 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}},\ c = 2.998 \times 10^8\ \mathrm{m/s},\ M_\odot = 1.989 \times 10^{30}\ \mathrm{kg}$.

2. Global Positioning System

The GPS navigation system uses a network of 24 satellites orbiting the Earth about 20.000 km above the ground with an orbital speed of 20.000 km/h. On board each satellite is an atomic clock that ticks with an accuracy of 1 nanosecond. A GPS receiver determines its current position by comparing the time signals it receives from several of the satellites and triangulating on the known positions of each. The receiver can determine your position to within 10 meters in only a few seconds. To achieve this level of precision, the clocks on the GPS satellites must be known to an accuracy of 30 nanoseconds. Determine whether it is necessary to take into account the kinematical- and the gravitational time dilation in the construction of this system. Calculate the error which is made in 24 hours by neglecting relativistic effects upon the rate of the satellite clocks.
Indication: $M_\oplus = 5.97 \times 10^{24}$ kg, $\bar{R}_\oplus = 6371$ km.

3. The flow of time : Interior solutions

1. Assume the Earth to be a perfect sphere with constant density. Compare the age of a rock in its center with the age of a rock in Mont Blanc. Which one is younger ? How much younger ? Is the weak field approximation appropriate ?
2. Consider an initial distribution of some radioactive element with a half-life of 3 billion years. Compare the present abundances of that element in the center and the surface of the Earth.
   Indication: The Earth is about 5×10^9 years old.

4. The Hafele-Keating experiment

The dilation of time was tested by Hafele and Keating in 1972 using cesium-beam atomic clocks transported on commercial flights around the Earth and comparing their reading on return to that on a standard clock in the surface of the Earth. The net effect is a combination of special relativistic effects and the gravitational change in the flow of time. The experiment was performed twice, one flying towards the east and one flying towards the west. The total flying time was 41.2 and 48.6 h respectively. Assuming the average speed and altitude of a commercial flights used in the Hafele-Keating experiment to be $v \simeq 300$ m/s and $h = 10 \pm 2$ km, estimate the time gain of the clock in the plane with respect to the one in Earth and compare it with the experimental result

	Eastward	Westward
$\Delta\tau_{\rm obs}$ (ns)	-59 ± 10	273 ± 7

5. Bending of light in the Newtonian framework

Suppose that photons are particles of infinitesimal mass m. Find the angle by which a photon is deviated by a small angle when it passes close to a spherical mass distribution. At infinity the photon propagates at the speed of light c. Apply the result to a photon that grazes the Sun. Find an approximate result.
$G = 6.673 \times 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}},\ c = 2.998 \times 10^{8}\ \mathrm{m/s},\ M_\odot = 1.989 \times 10^{30}\ \mathrm{kg}.$